Passivity based output observer selection for stable inverse

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This description is for MIMO LTI systems, LPV is a direct consequence of it.

1 Preliminary results from the literature

$$\Sigma : \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \xrightarrow{\text{state transformation}} \begin{cases} \dot{z} = q(z, y) \\ \dot{y} = a(z, y) + b(z, y)u \end{cases} \tag{1}$$

Theorem 1. Both relative degree and zero dynamics are invariant under feedback $u = \alpha(x) + \beta(x)v$. ◁

Theorem 2. Suppose $x = 0$ is a regular point for Σ.

$$\begin{array}{c} \Sigma \text{ is locally feedback eq. to a passive system} \\ \text{with } C^2 \text{ positive def. storage function } V \end{array} \Leftrightarrow \begin{array}{l} \Sigma \text{ has relative degree } \{1, .., 1\} \text{ at } x = 0 \\ \text{and is \textit{weekly} minimum phase} \end{array} \tag{2}$$

$$\begin{array}{l} \Sigma \text{ is \textit{globally} feedback eq. to a \textit{strictly} passive system} \\ \text{with } C^2 \text{ positive def. storage function } V \end{array} \Leftrightarrow \begin{array}{l} \Sigma \text{ has relative degree } \{1, .., 1\} \text{ at } x = 0 \\ \text{and is \textit{globally} minimum phase} \end{array} \tag{3}$$

◁

Definition 1. *Suppose $L_g h(0)$ is nonsingular. Than Σ is said to be* ***minimum phase*** *if $z = 0$ is an asymptotically stable equilibrium point of $\dot{z} = q(z, 0)$.* ◁

2 Dissipativity according to some supply function

2.1 Passivity of a semi-proper system

$$\begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx + Du \end{array}, \quad \begin{array}{l} V(x) = x^T P x \Rightarrow \dot{V}(x) = \begin{pmatrix} x^T & u^T \end{pmatrix} \begin{pmatrix} A^T P + PA & PB \\ B^T P & 0 \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} \\ \boxed{s(u, y) = u^T y + y^T u} = \begin{pmatrix} x^T & u^T \end{pmatrix} \begin{pmatrix} 0 & C^T \\ C & D + D^T \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} \end{array} \tag{4}$$

Dissipativity inequality:

$$\dot{V}(x) - s(u, y) \leq 0 \ \forall (x, u) \in \mathbb{R}^{n+m} \Leftrightarrow \begin{pmatrix} A^T P + PA & PB - C^T \\ B^T P - C & -D - D^T \end{pmatrix} \preceq 0 \tag{5}$$

Dissipativity inequality if $D = 0$ (i.e. $r \neq \{0, .., 0\}$):

$$\begin{pmatrix} A^T P + PA & PB - C^T \\ B^T P - C & 0 \end{pmatrix} \preceq 0, \quad \text{but } \not\prec 0 \quad \text{(it could be \textbf{only} negative \textbf{semi}-definite)} \tag{6}$$

Problémát jelent ez?

2.2 Strict output passivity of a semi-proper system

$$\begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx + Du \end{array}, \quad \begin{array}{l} V(x) = x^T P x \Rightarrow \dot{V}(x) = \begin{pmatrix} x^T & u^T \end{pmatrix} \begin{pmatrix} A^T P + PA & PB \\ B^T P & 0 \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} \\ \boxed{s(u, y) = u^T y + y^T u - y^T W y} = \begin{pmatrix} x^T & u^T \end{pmatrix} \begin{pmatrix} -C^T W C & C^T - C^T W D \\ C - D^T W C & D + D^T - D^T W D \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} \end{array} \tag{7}$$

Dissipativity inequality:

$$\dot{V}(x) - s(u,y) \leq 0 \ \forall (x,u) \in \mathbb{R}^{n+m} \Leftrightarrow \begin{pmatrix} A^T P + PA + C^T W C & PB - C^T + C^T W D \\ B^T P - C + D^T W C & -D - D^T + D^T W D \end{pmatrix} \preceq 0 \tag{8}$$

Applying Schur's complement lemma:

$$\begin{pmatrix} A^T P + PA & PB - C^T + C^T W D & C^T & 0 \\ B^T P - C + D^T W C & -D - D^T & 0 & D^T \\ C & 0 & -W^{-1} & 0 \\ 0 & D & 0 & -W^{-1} \end{pmatrix} \preceq 0 \ , \quad \begin{matrix} P \succ 0 \\ W \succ 0 \end{matrix} \tag{9}$$

Dissipativity inequality if $D = 0$ (i.e. $r \neq \{0,..,0\}$):

$$\begin{pmatrix} A^T P + PA & PB - C^T & C^T \\ B^T P - C & 0 & 0 \\ C & 0 & -W^{-1} \end{pmatrix} \preceq 0, \quad \text{but} \not\prec 0 \quad \text{(it could be **only** negative **semi-**definite)} \tag{10}$$

Problémát jelent ez?

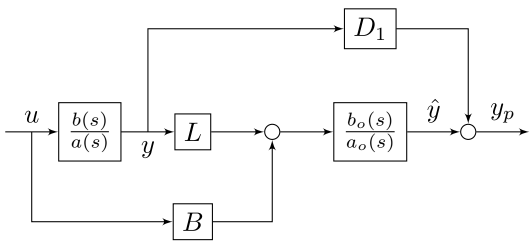
3 Facts from matrix algebra

1. Schur's complement lemma:

$$\begin{aligned} X = \begin{pmatrix} A & B \\ B^T & C \end{pmatrix} \succ 0 &\Leftrightarrow A \succ 0 \text{ and } C - B^T A^{-1} B \succ 0 \\ &\Leftrightarrow C \succ 0 \text{ and } A - B C^{-1} B^T \succ 0 \end{aligned} \tag{11}$$

   If $C = 0$, than X cannot be positive definite, since $-B^T A^{-1} B$ will be for sure negative definite for all $A \succ 0$.

4 Operátortartománybeli analízis



Ahol

$$\frac{b_o(s)}{a_o(s)} = C_1 \big(sI - A + LC \big)^{-1} \tag{12}$$

Szerintem ez MIMO esetben is jó ($a(s)$ és $a_o(s)$ skalár, minden egyéb mátrix):

$$\begin{aligned} y_p &= D_1 y + \hat{y} = \frac{1}{a(s)} D_1 b(s) u + \frac{1}{a_o(s)} b_o(s) \left(\frac{1}{a(s)} L b(s) + B \right) u \\ G_e(s) &= \frac{1}{a(s)} D_1 b(s) + \frac{1}{a_o(s)} b_o(s) \left(\frac{1}{a(s)} L b(s) + B \right) \\ G_e(s) &= \frac{1}{a(s) a_o(s)} D_1 b(s) a_o(s) + \frac{1}{a(s) a_o(s)} b_o(s) (L b(s) + B a(s)) \\ G_e(s) &= \frac{1}{a(s) a_o(s)} \Big(D_1 b(s) a_o(s) + b_o(s) L b(s) + b_o(s) B a(s) \Big) \end{aligned} \tag{13}$$

Tehát az új számláló:

$$b_e(s) = D_1 b(s) a_o(s) + b_o(s) L b(s) + b_o(s) B a(s) \tag{14}$$

A piros tagok lehetnek instabilak. Kék: szabad constans. Mivel $a_o(s)$ skalár:

$$b_e(s) = \Big(a_o(s) D_1 + b_o(s) L \Big) b(s) + b_o(s) B a(s) \leftarrow \text{ legyen stabil} \tag{15}$$

Numerikus példával illusztrálom, hogy ez valóban lehetséges:

$$\begin{aligned}
&G(s) = \frac{b(s)}{a(s)} = \frac{s-5}{s^2-3s+2}, \quad A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \ C = \begin{pmatrix} 0.5 & -2.5 \end{pmatrix} \\
&\text{observer: } L \overset{\texttt{place}}{:=} \begin{pmatrix} -13 \\ -5 \end{pmatrix}, \ C_1 := \begin{pmatrix} 1 & -1 \end{pmatrix} \Rightarrow G_o(s) = \frac{b_o(s)}{a_o(s)} = \frac{1}{s^2+3s+2} \begin{pmatrix} s+9 & -s-25 \end{pmatrix} \\
&b_e(s) = (D_1+2)s^3 + (4-2D_1)s^2 + (-13D_1-2)s - (10D_1+4)\Big|_{D_1=-1} \\
&\quad = s^3 + 6s^2 + 11s + 6 = (s+3)(s+2)(s+1) \\
&\text{eredő átviteli függvény: } G_e(s) = \frac{s+3}{s^2-3s+2}
\end{aligned} \tag{16}$$

Másik példa (ami a relatív degreet is megjavítja):

$$\begin{aligned}
&G(s) = \frac{s-5}{(s-3)(s^2-3s+2)}, \quad L^T \overset{\texttt{place}}{:=} \begin{pmatrix} -193 & -212 & -52 \end{pmatrix}, \ C_1 := \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}, \ D_1 := 0.1 \\
&G_e(s) = \frac{s^2+4.1s+3.5}{s^3-6s^2+11s-6}
\end{aligned} \tag{17}$$