Passivity based output observer selection for stable inverse

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version: July 3, 2017

This description is for MIMO LTI systems, LPV is a direct consequence of it.

1 Problem statement

There exists a LTI(LPV) system with unstable zeros and poles (no direct feedthrough for the moment being, $D = 0$). Our intention is to design a dynamic output passivisation controller (from about a passivisation signal r) downto a performance output y_p. We use dynamic principles to passivity. The over all closed loop system will be stable and passive, this hence means having stable zero dynamics. Consequently, a stable dynamic inversion w.r.t. any unknown inputs (faults, disturbance) can be created. This gives rise to a new paradigm in fault detection and estimation see 1.

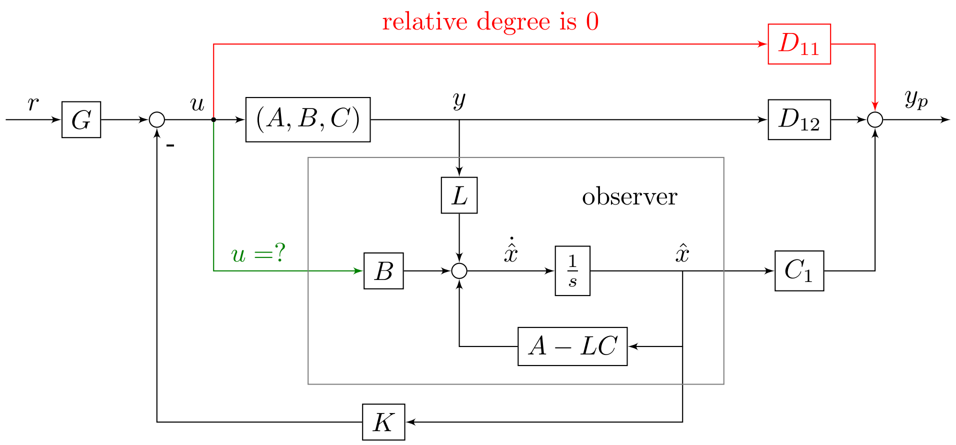


Figure 1: Block diagram for dynamic strict output passivisation

Goal $r \to y_p$ passive (strict, dynamic input)

Unknown G, K, L, C_1, D_{11}, D_{12}

Yield invertible/feedback linearisable later/ plant + observer

u, y are measured, y_p is performance output

Conclusion Any signal from r can be inverted

Fault r is considered as a fault input

2 Main result

Given

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$
$$y(t) = Cx(t) \tag{2}$$

Passivity based output feedback design for an LTI system is solved, where the observers outputs are selected such that the zero dynamics becomes stable. This enables stable dynamic inversion w.r.t. an input r, $\dim(u) = \dim(y)$. First a generalized state observer problem is considered then a Luenberger observer based one.

Generalized observer structure,

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{3}$$
$$y(t) = Cx(t), \tag{4}$$
$$u(t) = -K\hat{x}(t) + Gr(t) \tag{5}$$
$$\dot{\hat{x}}(t) = A_f\hat{x}(t) + B_{fr}u(t) + B_{fy}y(t) = (A_f - B_{fr}K)\hat{x}(t) + B_{fy}Cx(t) + B_{fr}Gr(t) \tag{6}$$
$$y_p(t) = C_f\hat{x}(t) + D_{fr}u(t) + D_{fy}y(t) = (C_f - D_{fr}K)\hat{x}(t) + D_{fy}Cx(t) + D_{fr}Gr(t) \tag{7}$$
$$\dot{X}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x(t) \\ \hat{x}(t) \end{pmatrix} = \begin{pmatrix} A & -BK \\ B_{fy}C & (A_f - B_{fr}K)) \end{pmatrix} \begin{pmatrix} x(t) \\ \hat{x}(t) \end{pmatrix} + \begin{pmatrix} BG \\ B_{fr}G \end{pmatrix} r(t) = \tilde{A}X(t) + \tilde{B}Gr(t) \tag{8}$$
$$y_p(t) = \begin{pmatrix} D_{fy}C & (C_f - D_{fr}K) \end{pmatrix} \begin{pmatrix} x(t) \\ \hat{x}(t) \end{pmatrix} + D_{fr}Gr(t) = \tilde{C}X(t) + \tilde{D}r(t) \tag{9}$$

Remark: error equation on state reconstruction $\tilde{x}(t) = x(t) - \hat{x}(t)$ has to be developed and may create a necessary condition for design. Note, $\dim(x) \neq \dim(\hat{x})$ might be an option!

Structured observer, Luenberger

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{10}$$
$$y(t) = Cx(t), \tag{11}$$
$$u(t) = -K\hat{x}(t) + Gr(t) \tag{12}$$
$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) = (A - LC - BK)\hat{x}(t) + LCx(t) + BGr(t) \tag{13}$$
$$\hat{y}(t) = C\hat{x}(t) \tag{14}$$
$$y_p(t) = C_1\hat{x}(t) + \underbrace{D_{11}u(t)}_{\text{Emiatt a relativ degree 0 lesz! Nem az a cél, hogy 1 legyen?}} + D_{12}y(t) = (C_1 - D_{11}K)\hat{x}(t) + D_{12}Cx(t) + D_{11}Gr(t) \tag{15}$$

$$\dot{X}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x(t) \\ \hat{x}(t) \end{pmatrix} = \begin{pmatrix} A & -BK \\ LC & A - LC - BK \end{pmatrix} \begin{pmatrix} x(t) \\ \hat{x}(t) \end{pmatrix} + \begin{pmatrix} BG \\ BG \end{pmatrix} r(t) = \tilde{A}X(t) + \tilde{B}r(t) \tag{16}$$
$$y_p(t) = \begin{pmatrix} D_{12}C & C_1 - D_{11}K \end{pmatrix} \begin{pmatrix} x(t) \\ \hat{x}(t) \end{pmatrix} + D_{11}Gr(t) = \tilde{C}X(t) + \tilde{D}r(t) \tag{17}$$

Remark: the error dynamics vanishes asymptotically since $\hat{y}(t) = C\hat{x}(t)$ and $A - LC$ being stable, i.e. $\lim_{t\to\infty} \tilde{x}(t) = 0$ with $\tilde{x}(t) = x(t) - \hat{x}(t)$.

$$\begin{aligned}
&K \in \mathbb{R}^{\dim(u)\times\dim(\hat{x})},\ L \in \mathbb{R}^{\dim(\hat{x})\times\dim(y)}, \qquad \dim(\hat{x}) \overset{\text{egyelőre}}{=} \dim(x) = n \\
&B \in \mathbb{R}^{\dim(x)\times\dim(u)},\ C \in \mathbb{R}^{\dim(y)\times\dim(x)} \\
&D_{11} \in \mathbb{R}^{\dim(y_p)\times\dim(u)},\ D_{12} \in \mathbb{R}^{\dim(y_p)\times\dim(y)} \\
&G \in \mathbb{R}^{\dim(u)\times\dim(r)} \\
&\tilde{B} = \left(\begin{smallmatrix} BG \\ BG \end{smallmatrix} \right) \in \mathbb{R}^{2\dim(x)\times\dim(r)} \\
&\tilde{C} = \left(\begin{smallmatrix} D_{12}C & C_1 - D_{11}K \end{smallmatrix} \right) \in \mathbb{R}^{\dim(y_p)\times 2\dim(x)} \\
&\tilde{D} = D_{11}G \in \mathbb{R}^{\dim(y_p)\times\dim(r)}, \qquad \dim(y_p) \overset{???}{=} \dim(r), \text{ gondolom ezért van } G\ldots
\end{aligned} \tag{18}$$

Proposition 1 *Stable dynamic inversion filter design by means of passivity*

Proof 1 *Strict output passivity condition for the closed loop reads as*

$$\dot{V}(X) \leq r(t)^T y_p(t) + y_p(t)^T r(t) - y_p(t)^T W y_p(t) \tag{19}$$

where with a given $W \succ 0$, $V(X) = X(t)^T P X(t)$, $P = P^T \succ 0$.

$$
\begin{aligned}
\text{l.h.s.}: \quad & \Big(\tilde{A}X(t) + \tilde{B}r(t)\Big)^T PX(t) + X(t)^T P\Big(\tilde{A}X(t) + \tilde{B}r(t)\Big) \\
& X(t)^T(\tilde{A}^T P + P\tilde{A})X(t) + r(t)^T \tilde{B}^T PX(t) + X(t)^T P\tilde{B}r(t) \\
& \begin{pmatrix} X(t) \\ r(t) \end{pmatrix}^T \begin{pmatrix} \tilde{A}^T P + P\tilde{A} & P\tilde{B} \\ \tilde{B}^T P & 0 \end{pmatrix} \begin{pmatrix} X(t) \\ r(t) \end{pmatrix} \\
\text{r.h.s.}: \quad & r(t)^T\Big(\tilde{C}X(t) + \tilde{D}r(t)\Big) + \Big(\tilde{C}X(t) + \tilde{D}r(t)\Big)^T r(t) - \Big(\tilde{C}X(t) + \tilde{D}r(t)\Big)^T W\Big(\tilde{C}X(t) + \tilde{D}r(t)\Big) \\
& r(t)^T\tilde{C}X(t) + X(t)^T\tilde{C}^T r(t) + r(t)^T(\tilde{D} + \tilde{D}^T)r(t) - \begin{pmatrix} X(t) \\ r(t) \end{pmatrix}^T \begin{pmatrix} \tilde{C}^T W\tilde{C} & \tilde{C}^T W\tilde{D} \\ \tilde{D}^T W\tilde{C} & \tilde{D}^T W\tilde{D} \end{pmatrix} \begin{pmatrix} X(t) \\ r(t) \end{pmatrix} \\
& \begin{pmatrix} X(t) \\ r(t) \end{pmatrix}^T \begin{pmatrix} -\tilde{C}^T W\tilde{C} & \tilde{C}^T - \tilde{C}^T W\tilde{D} \\ \tilde{C} - \tilde{D}^T W\tilde{C} & \tilde{D} + \tilde{D}^T - \tilde{D}^T W\tilde{D} \end{pmatrix} \begin{pmatrix} X(t) \\ r(t) \end{pmatrix}
\end{aligned} \tag{20}
$$

$$
M_0 = \begin{pmatrix} \tilde{A}^T P + P\tilde{A} + \tilde{C}^T W\tilde{C} & P\tilde{B} - \tilde{C}^T(I - W\tilde{D}) \\ \tilde{B}^T P - (I - \tilde{D}^T W)\tilde{C} & -\tilde{D} - \tilde{D}^T + \tilde{D}^T W\tilde{D} \end{pmatrix} \preceq 0 \tag{21}
$$

A piros tagra irom fel a Schur komplemenst.

$$
M_1 = \begin{pmatrix} \tilde{A}^T P + P\tilde{A} & P\tilde{B} - \tilde{C}^T(I - W\tilde{D}) & \tilde{C}^T \\ \tilde{B}^T P - (I - \tilde{D}^T W)\tilde{C} & -\tilde{D} - \tilde{D}^T + \tilde{D}^T W\tilde{D} & 0 \\ \tilde{C} & 0 & -W^{-1} \end{pmatrix} \preceq 0 \tag{22}
$$

This condition will remain bilinear since we co-design L, K *but can efficiently be solved though.*

$$
\widehat{M_1} := \begin{pmatrix} Q & & \\ & I & \\ & & I \end{pmatrix} M_1 \begin{pmatrix} Q & & \\ & I & \\ & & I \end{pmatrix} = \begin{pmatrix} Q\tilde{A}^T + \tilde{A}Q & \tilde{B} - Q\tilde{C}^T(I - W\tilde{D}) & Q\tilde{C}^T \\ \tilde{B}^T - (I - \tilde{D}^T W)\tilde{C}Q & -\tilde{D} - \tilde{D}^T + \tilde{D}^T W\tilde{D} & 0 \\ \tilde{C}Q & 0 & -W^{-1} \end{pmatrix} \preceq 0 \tag{23}
$$

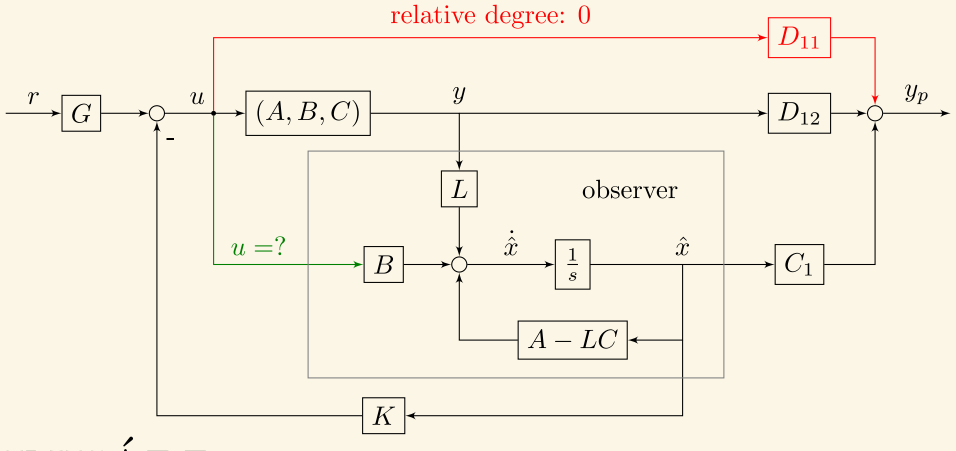
A Matlab kodban a piros tagok nem szerepelnek!
Ha a zold tagra is felirom a Schur komplemenst:

$$
M_2 = \begin{pmatrix} \tilde{A}^T P + P\tilde{A} & P\tilde{B} - \tilde{C}^T(I - W\tilde{D}) & \tilde{C}^T & 0 \\ \tilde{B}^T P - (I - \tilde{D}^T W)\tilde{C} & -\tilde{D} - \tilde{D}^T & 0 & \tilde{D}^T \\ \tilde{C} & 0 & -W^{-1} & 0 \\ 0 & \tilde{D} & 0 & -W^{-1} \end{pmatrix} \preceq 0, \tag{24}
$$

$$
\widehat{M_2} = \begin{pmatrix} Q\tilde{A}^T + \tilde{A}Q & \tilde{B} - Q\tilde{C}^T(I - W\tilde{D}) & Q\tilde{C}^T & 0 \\ \tilde{B}^T - (I - \tilde{D}^T W)\tilde{C}Q & -\tilde{D} - \tilde{D}^T & 0 & \tilde{D}^T \\ \tilde{C}Q & 0 & -W^{-1} & 0 \\ 0 & \tilde{D} & 0 & -W^{-1} \end{pmatrix} \preceq 0, \tag{25}
$$

where

$$
\begin{aligned}
\tilde{A} &= \begin{pmatrix} A & -BK \\ LC & A - LC - BK \end{pmatrix}, & \tilde{B} &= \begin{pmatrix} BG \\ BG \end{pmatrix} \\
\tilde{C} &= \begin{pmatrix} D_{12}C & C_1 - D_{11}K \end{pmatrix}, & \tilde{D} &= D_{11}G
\end{aligned} \tag{26}
$$



INKÁBB, let W be a fixed diagonal matrix, furthermore, we co-design L and K. Let the free decision variables of the bilinear problem be:

$$\begin{aligned} &Q,\ K,\ L,\ G,\ \tilde{D} \\ &N = Q\tilde{C}^T \in \mathbb{R}^{2\dim(x)\times\dim(y_p)} \end{aligned} \tag{27}$$

Furthermore, we define the following matrices, which depend linearly on the free variables:

$$\begin{aligned} \tilde{A} &= \begin{pmatrix} A & -BK \\ LC & A - LC - BK \end{pmatrix} \\ \tilde{B} &= \begin{pmatrix} BG \\ BG \end{pmatrix},\ G \neq 0 \end{aligned} \tag{28}$$

Then, the bilinear problem is:

$$\widehat{M}_2 = \begin{pmatrix} Q\tilde{A}^T + \tilde{A}Q & \tilde{B} - N + NW\tilde{D} & N & 0 \\ \tilde{B}^T - N^T + \tilde{D}^T W N^T & -\tilde{D} - \tilde{D}^T & 0 & \tilde{D}^T \\ N^T & 0 & -W^{-1} & 0 \\ 0 & \tilde{D} & 0 & -W^{-1} \end{pmatrix} \preceq 0 \quad \text{(BMI)}, \tag{29}$$

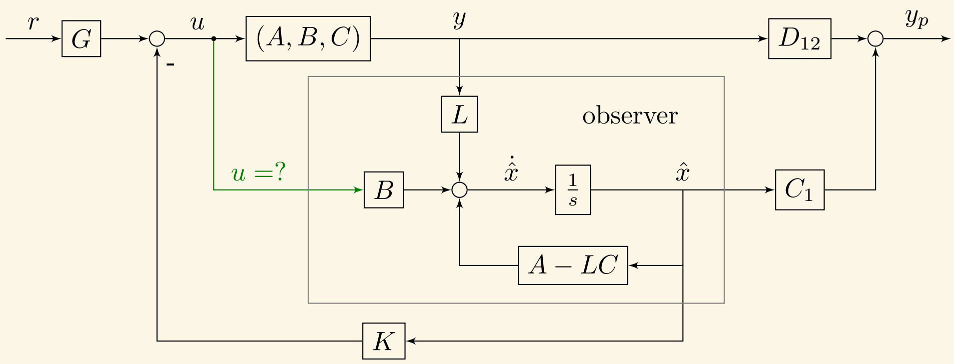
$$Q \succeq 0, \quad G \neq 0^{***} \tag{30}$$

Majd, ha ezt megoldottam:

$$\begin{aligned} &\tilde{C} = (Q^{-1}N)^T \\ &\tilde{D} = D_{11}G \Rightarrow D_{11} = \tilde{D}G^{-1} \\ &\tilde{C} := \begin{pmatrix} \tilde{C}_1 & \tilde{C}_2 \end{pmatrix} = \begin{pmatrix} D_{12}C & C_1 - D_{11}K \end{pmatrix} \\ &\text{ezért: } \begin{cases} \tilde{C}_1 = D_{12}C & \Rightarrow \quad D_{12} = \tilde{C}_1 C^T \left(CC^T\right)^{-1} \\ \tilde{C}_2 = C_1 - D_{11}K & \Rightarrow \quad C_1 = \tilde{C}_2 + D_{11}K \end{cases} \end{aligned} \tag{31}$$

*** Ezt kellene valahogy értelmesen megfogalmazni. SISO esetben egyszerű: $G > 0$.

Ha $D_{11}=0$, azaz $\tilde{D}=0$



Let W be a fixed diagonal matrix, furthermore, we co-design L and K.
Let the free decision variables of the bilinear problem be:

$$\begin{aligned} &Q,\ K,\ L,\ G \\ &N = Q\tilde{C}^T \in \mathbb{R}^{2\dim(x)\times\dim(y_p)} \end{aligned} \tag{32}$$

Furthermore, we define the following matrices, which depend linearly on the free variables:

$$\begin{aligned} \tilde{A} &= \begin{pmatrix} A & -BK \\ LC & A-LC-BK \end{pmatrix} \\ \tilde{B} &= \begin{pmatrix} BG \\ BG \end{pmatrix},\ G \neq 0 \end{aligned} \tag{33}$$

Then, the bilinear problem is:

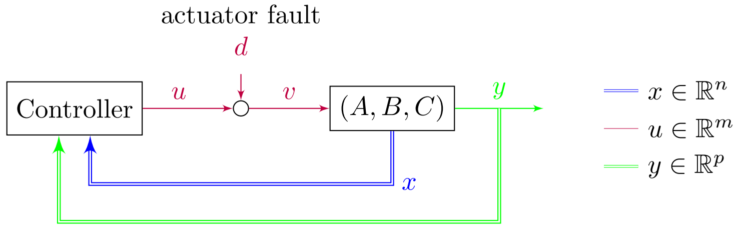
$$\widehat{M}_2 = \begin{pmatrix} Q\tilde{A}^T + \tilde{A}Q & \tilde{B}-N & N \\ \tilde{B}^T - N^T & 0 & 0 \\ N^T & 0 & -W^{-1} \end{pmatrix} \preceq 0 \quad \text{(BMI – ez kapasbol indefinite!)}, \tag{34}$$

$$Q \succeq 0, \quad G \neq 0 \tag{35}$$

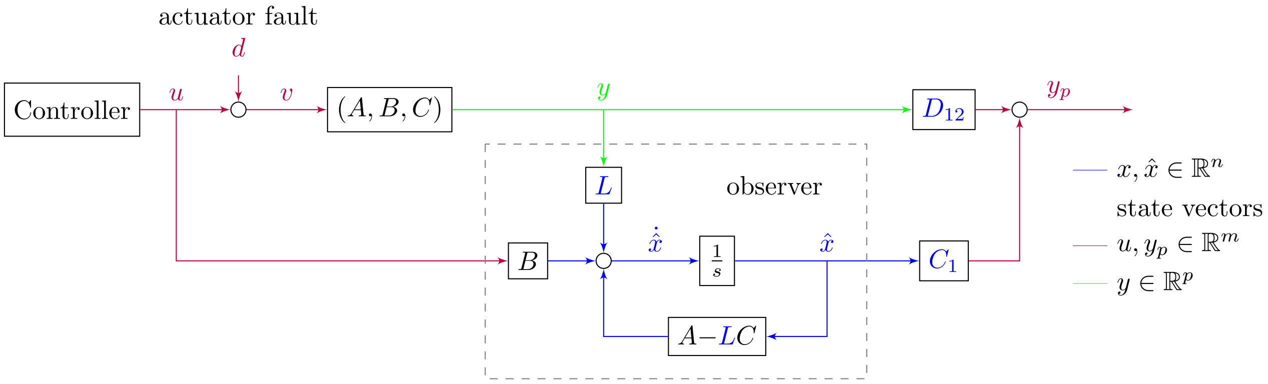
Majd, ha ezt megoldottam:

$$\begin{aligned} &\tilde{C} = (Q^{-1}N)^T \\ &\tilde{C} := \begin{pmatrix}\tilde{C}_1 & \tilde{C}_2\end{pmatrix} = \begin{pmatrix} D_{12}C & C_1 \end{pmatrix} \\ &\text{ezért: } \begin{cases} \tilde{C}_1 = D_{12}C & \Rightarrow \quad D_{12} = \tilde{C}_1 C^T \left(CC^T\right)^{-1} \\ \tilde{C}_2 = C_1 \end{cases} \end{aligned} \tag{36}$$

3 MIMO, $u \to y_p$ passivization, feedback equivalence with a passive system



Having an LTI MIMO system, which is somehow fed back through a controller (either tuned by output vector or by the full state vector). The actuator is assumed to be faulty. We intend to detect its fault using system inversion.]. However, the system is not invertible, since its zeros are unstable and/or its vector relative degree (v.r.d.) is more than 1, therefore, we augment the system with an additional (linear) dynamics, which is tuned by the system's output y and the designed control input u, hence its resemblance to an observer.



$$\begin{cases} \dot{x} = Ax + Bv \\ y = Cx \end{cases} \quad \begin{cases} \dot{\hat{x}} = LCx + (A - LC)\hat{x} + Bu \\ y_p = D_{12}Cx + C_1\hat{x} \end{cases} \Rightarrow \begin{cases} \dot{\xi} = \begin{pmatrix} A & 0 \\ LC & A - LC \end{pmatrix} \xi + \begin{pmatrix} B \\ B \end{pmatrix} u + \begin{pmatrix} B \\ 0 \end{pmatrix} d \\ y_p = \begin{pmatrix} D_{12}C & C_1 \end{pmatrix} \xi, \quad \text{where } \xi := \begin{pmatrix} x \\ \hat{x} \end{pmatrix} \end{cases} \quad \begin{array}{l} \text{dimensions:} \\ B \in \mathbb{R}^{n\times m} \\ C \in \mathbb{R}^{p\times n} \\ C_1 \in \mathbb{R}^{m\times n} \\ D_{12} \in \mathbb{R}^{p\times m} \end{array} \tag{37}$$

Proposition 1. (Vector relative degree) *LTI MIMO*

If matrix B is full column rank, there exists matrix C_1 such that the system $u \to y_p$ will have v.r.d. 1.

Proof.

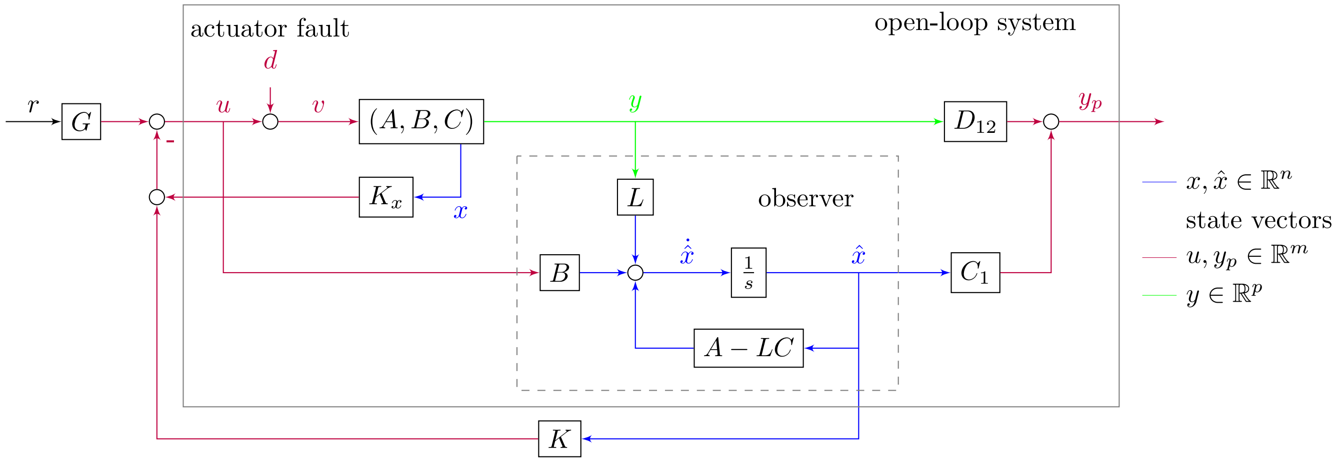
$$\begin{aligned} \dot{y}_p &= \begin{pmatrix} D_{12}C & C_1 \end{pmatrix} \begin{pmatrix} A & 0 \\ LC & A - LC \end{pmatrix} \xi + \begin{pmatrix} D_{12}C & C_1 \end{pmatrix} \begin{pmatrix} B \\ B \end{pmatrix} u + D_{12}CBd \\ &= \begin{pmatrix} D_{12}CA + C_1LC & C_1(A - LC) \end{pmatrix} \xi + \Big(\underbrace{D_{12}CB}_{\text{could be rank deficient}} + C_1B\Big) u + D_{12}CBd \end{aligned} \tag{38}$$

Let $R \in \mathbb{R}^{m\times m}$ be full rank matrix. Then, for $C_1B = R - D_{12}CB$, we have infinitely many solutions, since it is an under-determinant system of linear equations in C_1 unknown. The least Frobenius norm solution for C_1 is:

$$C_1 = (R - D_{12}CB)(B^TB)^{-1}B^T \tag{39}$$

Since B is full column rank, $B^TB \in \mathbb{R}^{m\times m}$ is invertible, thus, $D_{12}CB + C_1B = R$ is a rank matrix. □

The goal is to chose matrices L, C_1, D_{12} such that the system $u \to y_p$ be feedback equivalent to a passive system and the v.r.d. of $u \to y_p$ be 1.



In other words, we need to find matrices K_x, K, G, such that the closed loop system is passive.

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{40}$$

$$u = -K_x x - K\hat{x} + Gr \tag{41}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) = (A - LC - BK)\hat{x} + (LC - BK_x)x + BGr \tag{42}$$

$$y_p = C_1\hat{x} + D_{12}y = D_{12}Cx + C_1\hat{x} \tag{43}$$

$$\begin{aligned} \tilde{A} &= \begin{pmatrix} A - BK_x & -BK \\ LC - BK_x & A - LC - BK \end{pmatrix}, \quad \tilde{B} = \begin{pmatrix} BG \\ BG \end{pmatrix} \\ \tilde{C} &= \begin{pmatrix} D_{12}C & C_1 \end{pmatrix}, \quad G \neq 0 \end{aligned} \tag{44}$$

Let W be a fixed diagonal matrix, furthermore, we co-design L and K. The free decision variables of the bilinear problem are:

$$\begin{aligned} &Q,\ K,\ K_x,\ L,\ G \\ &N = Q\tilde{C}^T \in \mathbb{R}^{2\dim(x)\times\dim(y_p)} \end{aligned} \tag{45}$$

Furthermore, $\tilde{A}$ and $\tilde{B}$ are matrices, which depend linearly on these free variables. The bilinear constraint is:

$$\widehat{M_2} = \begin{pmatrix} Q\tilde{A}^T + \tilde{A}Q & \tilde{B} - N & N \\ \tilde{B}^T - N^T & 0 & 0 \\ N^T & 0 & -W^{-1} \end{pmatrix} \preceq 0 \quad \text{(BMI)}, \tag{46}$$

$$\text{while} \quad Q \succeq 0, \quad G \neq 0 \tag{47}$$

After we solved the bilinear problem, we can give the value of $\tilde{C}$, and hence of D_{12} and C_1:

$$\begin{aligned} &\tilde{C} = (Q^{-1}N)^T \\ &\tilde{C} := \begin{pmatrix} \tilde{C}_1 & \tilde{C}_2 \end{pmatrix} = \begin{pmatrix} D_{12}C & C_1 \end{pmatrix} \\ &\text{thus:} \begin{cases} \tilde{C}_1 = D_{12}C & \Rightarrow \quad D_{12} = \tilde{C}_1 C^T \left(CC^T\right)^{-1} \quad \text{if } \mathrm{Im}(\mathrm{C}^\mathrm{T}) \overset{!!!}{\subseteq} \mathrm{Im}(\tilde{\mathrm{C}}_1^\mathrm{T}) \\ \tilde{C}_2 = C_1 \end{cases} \end{aligned} \tag{48}$$

Matrices of the open loop system are

$$\begin{aligned} \mathcal{A} &= \begin{pmatrix} A & 0 \\ LC & A - LC \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} B \\ B \end{pmatrix} \\ \mathcal{C} &= \begin{pmatrix} D_{12}C & C_1 \end{pmatrix} \end{aligned} \tag{49}$$

System $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ is feedback equivalent with a passive system, therefore, it can be stabilized by a static output feedback.

3.1 Numerical example – MIMO

Having an LTI system with matrices:

$$A = \begin{pmatrix} 6.1 & 1.4 & -0.33 & -1.8 & 0.88 & -0.88 \\ -3.2 & 0.5 & 0.27 & 0.14 & -0.068 & 0.068 \\ 1.4 & -1.5 & 0.58 & 0.14 & -0.07 & 0.07 \\ -0.76 & -1.2 & -0.13 & 0.85 & 1.1 & -1.1 \\ 0.38 & 0.61 & 0.067 & 2.7 & 3.6 & -1.6 \\ -0.38 & -0.61 & -0.067 & 0.59 & 1.7 & 0.3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1.6 & 1 \\ 0.82 & 2 \\ -0.82 & 0 \end{pmatrix}$$
$$C = \begin{pmatrix} -0.15 & -0.24 & -0.026 & -0.2 & 0.6 & -0.6 \\ 0.39 & 0.17 & -1 & 0.58 & 0.21 & -1.2 \end{pmatrix} \tag{50}$$

Its transfer function and its zeros are:

$$H(s) = \begin{pmatrix} \dfrac{s-1}{(s+1)(s-2)} & \dfrac{1}{s-3} \\[2ex] \dfrac{(s+2)(s-7)}{(s-5)(s-1)(s-2)} & \dfrac{s-6}{(s-2)(s-3)} \end{pmatrix}$$
$$\texttt{tzero(H)} = \begin{pmatrix} 7.2854 & 0.0214 & 3 & 2 & 1.8361 \end{pmatrix} \tag{51}$$

Nem értem, MIMO-ba hogy számoljuk a zérusokat?

This system has a v.r.d. 1, since

$$\dot{y} = CAx + CBu, \quad CB = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{52}$$

After the optimizations, we obtained the following matrices:
VER1:

$$\boxed{\begin{aligned} L^T &= \begin{pmatrix} -12.1 & -7.94 & 4.96 & 12.7 & 13.4 & 9.3 \\ 28 & 13.9 & -5.47 & -11.2 & 10.5 & -19.4 \end{pmatrix}, \quad \text{PROBLEM: } A - LC \text{ not stable} \\ C_1 &= \begin{pmatrix} 0.696 & -0.394 & 0.249 & -0.222 & -0.0848 & -0.0626 \\ 0.185 & 0.377 & -0.126 & 0.772 & 0.825 & 0.699 \end{pmatrix}, \quad D_{12} = \begin{pmatrix} 0.0249 & -0.0004 \\ 0.829 & -0.357 \end{pmatrix} \end{aligned}} \tag{53}$$

$$K_x = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{it was constrained to be 0, still feasible}$$
$$K = \begin{pmatrix} 43.1 & -19.8 & 16 & -10.2 & 2.67 & 2.29 \\ -11.7 & 2.26 & 3.52 & 6.51 & 25.1 & 13.9 \end{pmatrix}, \quad G = \begin{pmatrix} 4.2 & 0.0988 \\ 0.0988 & 5.95 \end{pmatrix} \tag{54}$$

VER2:

$$L^T = \begin{pmatrix} -9.45 & -82.2 & 206 & 57.3 & -193 & -199 \\ -85.7 & 58.7 & -50.2 & -0.158 & 16 & 16 \end{pmatrix}, \quad \text{designed previously using } \texttt{place}$$
$$C_1 = \begin{pmatrix} -0.752 & -2.67 & 2.99 & 1.59 & -0.042 & 1.45 \\ 2.67 & 5.03 & -6.46 & -3.4 & 1.67 & 2.02 \end{pmatrix}, \quad D_{12} = \begin{pmatrix} 1.09 & 0.21 \\ 5.02 & -3.83 \end{pmatrix} \tag{55}$$

$$K_x = \begin{pmatrix} 165 & -101 & 99.5 & -32.7 & 20.4 & -60.3 \\ 71.9 & -18.2 & 0.115 & 52.1 & 154 & 154 \end{pmatrix}$$
$$K = \begin{pmatrix} -2.83 & -49.4 & 58.3 & 47.5 & -13.9 & 50.8 \\ 7.53 & 5.74 & -25.3 & -42.1 & 63.6 & 17.2 \end{pmatrix}, \quad G = \begin{pmatrix} 7.43 & -1.23 \\ -1.23 & 7.47 \end{pmatrix} \tag{56}$$

Problem: the augmented open loop system is not minimum phase! (The constraints are not precisely fulfilled.)