Fault detection of MIMO LTI systems using output passivisation with Luenberger dynamic observer design

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Abstract

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1 Introduction

1.1 Problem statement

There exists an LTI system with unstable zeros and poles (no direct feedthrough for the moment being, $D = 0$). Our intention is to design a dynamic output passivisation controller (from about a passivisation signal r) downto a performance output y_p. We use dynamic principles to passivity. The over all closed loop system will be stable and passive, this hence means having stable zero dynamics. Consequently, a stable dynamic inversion w.r.t. any unknown inputs (faults, disturbance) can be created. This gives rise to a new paradigm in fault detection and estimation see **??**.

2 MIMO, $v \to y_p$ passivisation, feedback equivalence with a passive system

Having an LTI MIMO system, which is somehow fed back through a controller (either tuned by output vector or by the full state vector). The actuator is assumed to be faulty. We intend to detect its fault using system inversion. However, the system is not invertible, since its zeros are unstable and/or its relative degree is not $\{1, ..., 1\}$, therefore, we augment the system with an additional (linear) dynamics, which is tuned by the system's output y and the designed control input u (Luenberger observer). The goal is to chose matrices L, C_1, D_1 such that the system $v \to y_p$ be feedback equivalent to a passive system with relative relative degree $r = \{1, ..., 1\}$.

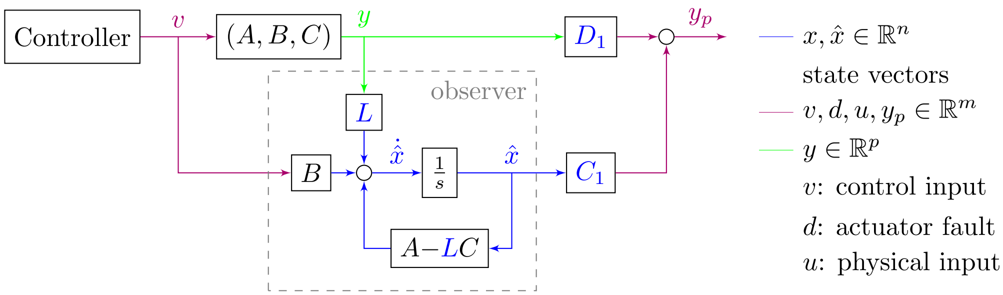


Figure 1: Block diagram of the augmented (open-loop) system with state observer without actuator fault.

Proposition 1. If there is no actuator fault ($d(t) = 0$), signal $\hat{v}(t)$ can be reconstructed from signals y_p and $\hat{x}$, such that $\hat{v}(t)$ converges exponentially to $v(t)$.

Proof. The equation of the original system and the Luenberger state observer with artificial output y_p are

$$\begin{cases} \dot{x} = Ax + Bv \\ y = Cx \end{cases} \qquad \begin{cases} \dot{\hat{x}} = (A - LC)\hat{x} + Bv + LCx \\ y_p = D_1 Cx + C_1 \hat{x} \end{cases} \tag{1}$$

Introducing the new state vector $X = \left(\begin{smallmatrix} x \\ \hat{x} \end{smallmatrix}\right)$, the equation of the augmented system $v \to y_p$ is

$$\begin{cases} \dot{X} = \widehat{A}X + \widehat{B}v \\ y_p = \widehat{C}X \end{cases} \quad \text{where} \quad \widehat{C} = \begin{pmatrix} D_1 C & C_1 \end{pmatrix}, \; \widehat{A} = \begin{pmatrix} A & 0 \\ LC & A - LC \end{pmatrix}, \; \widehat{B} = \begin{pmatrix} B \\ B \end{pmatrix} \tag{2}$$

The derivative of the artificial output is

$$\begin{aligned} \dot{y}_p = \widehat{C}\widehat{A}X + \widehat{C}\widehat{B}v = \begin{pmatrix} D_1 C & C_1 \end{pmatrix} \begin{pmatrix} A & 0 \\ LC & A - LC \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix} + \begin{pmatrix} D_1 C & C_1 \end{pmatrix} \begin{pmatrix} B \\ B \end{pmatrix} v \\ = \big(D_1 CA + C_1 LC\big)x + \big(C_1 A - C_1 LC\big)\hat{x} + \big(D_1 C + C_1\big)Bv \\ = \big(D_1 CA + C_1 LC\big)e + \big(D_1 CA + \underset{\sim\sim}{C_1 LC}\big)\hat{x} + \big(C_1 A \underset{\sim\sim}{- C_1 LC}\big)\hat{x} + \big(D_1 C + C_1\big)Bv \\ = \big(D_1 CA + C_1 LC\big)e + \big(D_1 CA + C_1 A\big)\hat{x} + \big(D_1 C + C_1\big)Bv \end{aligned} \tag{3}$$

Since $r = \{1, ..., 1\}$, $\widehat{C}\widehat{B} = \big(D_1 C + C_1\big)B$ is non-singular and hence invertible, therefore, the control input can be explicitly expressed as:

$$v = \underbrace{M\dot{y}_p - M\big(D_1 CA + C_1 A\big)\hat{x}}_{\text{can be computed}} - \underbrace{M\big(D_1 CA + C_1 LC\big)e}_{e \text{ is unknown}}, \quad \text{where } M = \Big[\big(D_1 C + C_1\big)B\Big]^{-1} \tag{4}$$

In case of a stable observer, the observation error $e = x - \hat{x}$ tends exponentially to zero, hence

$$\hat{v} := M\dot{y}_p - M\big(D_1 CA + C_1 A\big)\hat{x} \quad \Rightarrow \quad e_v = v - \hat{v} = -M\big(D_1 CA + C_1 LC\big)e \to 0 \tag{5}$$

Tends exponentially to v, which completes the proof. During this derivation, we used only the relative degree $r = \{1, ..., 1\}$ property of the system, passivity is not required yet. □

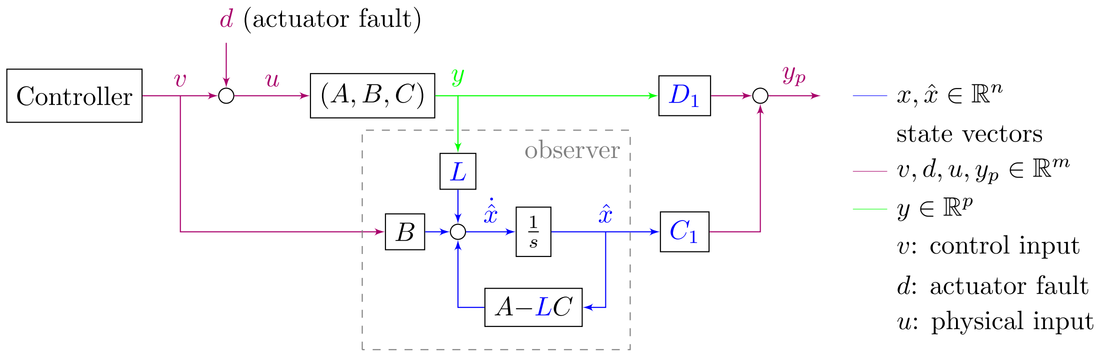


Figure 2: Block diagram of the augmented (open-loop) system with state observer without actuator fault.

Proposition 2. If there **is** actuator fault ($d(t) \neq 0$), residual signal $\hat{v}(t) - v(t)$ will not vanish. Residual signal $\hat{v}(t) - v(t)$ can be constructed using the

- ▷ observed state vector $\hat{x}(t)$,
- ▷ artificial output $y_p(t)$,
- ▷ designed control input $v(t)$.

Proof. The equation of the original system and the Luenberger state observer with artificial output y_p are

$$\begin{cases} \dot{x} = Ax + Bv + Bd \\ y = Cx \end{cases} \qquad \begin{cases} \dot{\hat{x}} = (A - LC)\hat{x} + Bv + LCx \\ y_p = D_1Cx + C_1\hat{x} \end{cases} \tag{6}$$

Introducing the new state vector $X = \left(\begin{smallmatrix} x \\ \hat{x} \end{smallmatrix}\right)$, the equation of the augmented system $v \to y_p$ is

$$\begin{cases} \dot{X} = \widehat{A}X + \widehat{B}v + \widehat{E}d \\ y_p = \widehat{C}X \end{cases}, \text{ where } \widehat{C} = \begin{pmatrix} D_1C & C_1 \end{pmatrix}, \ \widehat{A} = \begin{pmatrix} A & 0 \\ LC & A - LC \end{pmatrix}, \ \widehat{B} = \begin{pmatrix} B \\ B \end{pmatrix}, \ \widehat{E} = \begin{pmatrix} B \\ 0 \end{pmatrix} \tag{7}$$

The derivative of the artificial output is

$$\dot{y}_p = \widehat{C}\widehat{A}X + \widehat{C}\widehat{B}v + \widehat{C}\widehat{E}d = \big(D_1CA + C_1LC\big)e + \big(D_1CA + C_1A\big)\hat{x} + \big(D_1C + C_1\big)Bv + D_1CBd \tag{8}$$

Since $r = \{1, ..., 1\}$, $\widehat{C}\widehat{B} = \big(D_1C + C_1\big)B$ is non-singular and hence invertible, therefore, the control input can be explicitly expressed as:

$$v = M\dot{y}_p - M\big(D_1CA + C_1A\big)\hat{x} - M\big(D_1CA + C_1LC\big)e - MD_1CBd, \quad \text{where } M = \Big[\big(D_1C + C_1\big)B\Big]^{-1} \tag{9}$$

Since the observation error $e(t)$ and the actuator fault $d(t)$ is unknown, the original control input signal can be estimated as follows

$$\hat{v} = M\dot{y}_p - M\big(D_1CA + C_1A\big)\hat{x} \tag{10}$$

If the actuator fault input is non-zero, the residual signal

$$-e_v(t) = \hat{v}(t) - v(t) = \underbrace{M\big(D_1CA + C_1LC\big)e(t)}_{\to 0} + MD_1CBd(t) \tag{11}$$

will not vanish, unless $d(t) \in \mathrm{Ker}\big(MD_1CB\big)$. □

Proposition 3. (Vector relative degree – a feasibility study for $r = \{1, ..., 1\}$) LTI MIMO
If matrix B is full column rank, $\exists C_1$ such that the system $u \to y_p$ will have relative degree $r = \{1, .., 1\}$.

Proof.

$$\begin{aligned}\dot{y}_p &= \begin{pmatrix} D_1C & C_1 \end{pmatrix} \begin{pmatrix} A & 0 \\ LC & A - LC \end{pmatrix} X + \begin{pmatrix} D_1C & C_1 \end{pmatrix} \begin{pmatrix} B \\ B \end{pmatrix} u + D_1CBd \\ &= \begin{pmatrix} D_1CA + C_1LC & C_1(A - LC) \end{pmatrix} X + \Big(\underbrace{D_1CB}_{\text{could be rank deficient}} + C_1B\Big) u + D_1CBd \end{aligned} \tag{12}$$

Let $R \in \mathbb{R}^{m \times m}$ be full rank matrix. Then, for $C_1B = R - D_1CB$, we have infinitely many solutions, since it is an under-determinant system of linear equations in C_1 unknown. The least Frobenius norm solution for C_1 is:

$$C_1 = (R - D_1CB)(B^TB)^{-1}B^T \tag{13}$$

Since B is full column rank, $B^TB \in \mathbb{R}^{m \times m}$ is invertible, thus, $D_1CB + C_1B = R$ is a rank matrix. □

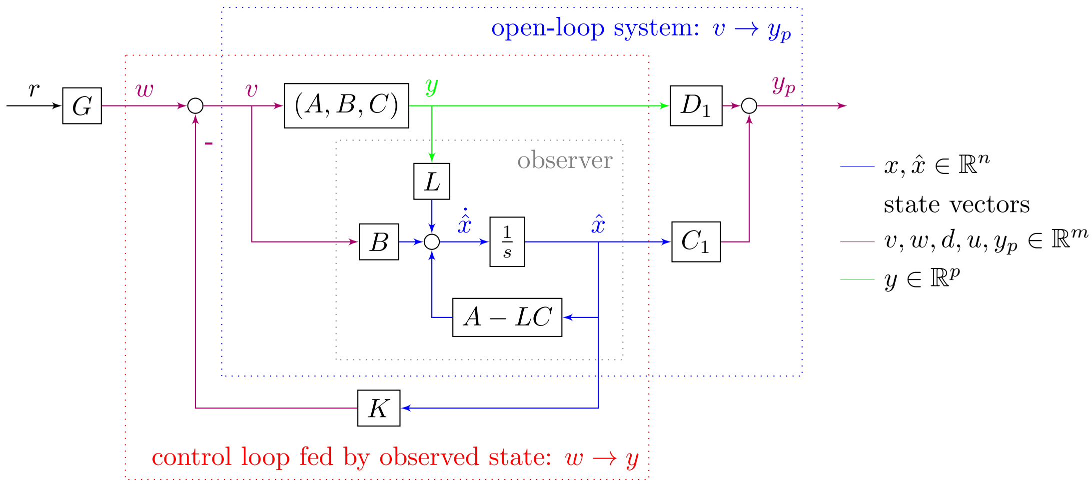


Figure 3: Passive state feedback equivalent of the augmented model.

Observer filter design for model inversion using optimization (no actuator fault)

In this section, the model of the augmented system is built differently compared Eq. (2). The state vector of model is chosen to be $\xi = \left(\begin{smallmatrix} x \\ e \end{smallmatrix}\right)$. According to Eq. (1), the error dynamics is

$$\dot{e} = (A - LC)e, \tag{14}$$

therefore, the dynamics of the augmented open-loop system (Figure 3, $v \to y_p$) can be given as follows:

$$\begin{cases} \dot{\xi} = \widetilde{A}\xi + \widetilde{B}v \\ y_p = \widetilde{C}\xi \end{cases}, \quad \text{where } \widetilde{A} = \begin{pmatrix} A & 0 \\ 0 & A - LC \end{pmatrix}, \ \widetilde{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \ \widetilde{C} = \begin{pmatrix} D_1 C + C_1 & -C_1 \end{pmatrix} \tag{15}$$

The dynamics of the feedback equivalent of the augmented open-loop system (Figure 3, $r \to y_p$) is the following (subscripts "c" stand for "closed-loop"):

$$\begin{cases} \dot{\xi} = \widetilde{A}_c\xi + \widetilde{B}_c r \\ y_p = \widetilde{C}\xi \end{cases}, \quad \text{where } \widetilde{A}_c = \begin{pmatrix} A - BK & BK \\ 0 & A - LC \end{pmatrix}, \ \widetilde{B}_c = \begin{pmatrix} BG \\ 0 \end{pmatrix} \tag{16}$$

Proposition 4. (Feedback equivalence of (15) and (16)) According to definition presented in [1], system $v \to y_p$ (15) is feedback equivalent with system $r \to y_p$ (16), since there exist $\alpha(\xi) = -K\hat{x}$ and $\beta(\xi) = G$ functions, such that the input $v = \alpha(\xi) + \beta(x)r$ applied to system $v \to y_p$ generates system $r \to y_p$.

In the sense of principle of separation of estimation and control, we can design a stabilizing state feedback gain K and a stable observer with gain L independently, which both gives a stable control loop fed by observed state vector (Figure 3, $w \to y$). Since the value of K is irrelevant in the model inversion point of view, it can be design arbitrarily before the optimization (eg. pole placement), matrix $G = I_m$ is chosen to be the identity matrix. During the optimization, we search for appropriate values for matrices C_1, D_1 and L (being free variables of the optimization), such that the closed-loop system $r \to y_p$ (the feedback equivalent of $v \to y_p$) be strict output passive, meaning that the following inequality must hold:

$$\dot{V}(\xi) \le r^T y_p + y_p^T r - y_p^T W y_p, \tag{17}$$

where

$$V(\xi) = \xi^T P \xi^T, \quad P = P^T \in \mathbb{R}^{2n \times 2n} \tag{18}$$

is a quadratic storage function for system $r \to y_p$. Inequality (17) can be developed further into the form:

$$s(\xi, r) := \xi^T \left(\widetilde{A}^T P + P\widetilde{A}\right)\xi + \xi^T P\widetilde{B}_c r - r^T \widetilde{C}\xi - \xi^T \widetilde{C}^T r + \xi^T \widetilde{C}^T W \widetilde{C}\xi \leq 0 \tag{19}$$

which is equivalent to the following matrix inequality

$$s(\xi, r) = \begin{pmatrix} \xi \\ r \end{pmatrix}^T \Lambda_1 \begin{pmatrix} \xi \\ r \end{pmatrix} \leq 0 \quad \Leftrightarrow \quad \Lambda_1 = \begin{pmatrix} \widetilde{A}^T P + P\widetilde{A} + \widetilde{C}^T W \widetilde{C} & P\widetilde{B}_c - \widetilde{C}^T \\ \widetilde{B}_c^T P - \widetilde{C} & 0 \end{pmatrix} \preceq 0 \tag{20}$$

In order to linearise (20), the Lyapunov matrix P is constrained to be block diagonal

$$P := \begin{pmatrix} Q & 0 \\ 0 & S \end{pmatrix} \succ 0, \quad P \in \mathbb{R}^{2n \times 2n}, \tag{21}$$

Furthermore, if we introduce $N := SL$, the bilinear expression in the first element of matrix Λ_1 can be evaluated further as follows:

$$\begin{aligned} \widetilde{A}^T P + P\widetilde{A} &= \begin{pmatrix} Q(A - BK) + (A - BK)^T Q & QBK \\ K^T B^T Q & SA + A^T S - SLC - C^T L^T S \end{pmatrix} \\ &= \begin{pmatrix} Q(A - BK) + (A - BK)^T Q & QBK \\ K^T B^T Q & SA + A^T S - NC - C^T N^T \end{pmatrix}, \end{aligned} \tag{22}$$

which is a linear expression in the matrix valued indeterminates Q, S and N. Since S is invertible ($S \succ 0$), matrix L can be reconstructed in the knowledge of matrices N and S after the optimization.

2.1 Numerical results (in case of $H_2(s)$ – stable)

We design a stabilizing feedback gain K for the original system ($u = -Kx$):

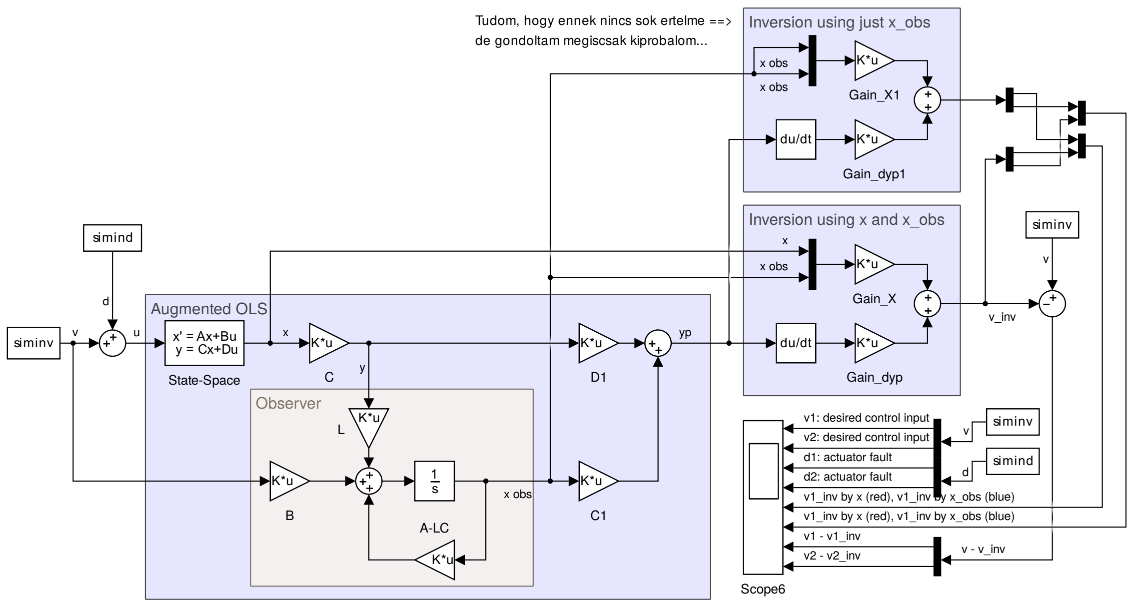
$$\begin{aligned} &p = \begin{pmatrix} -2 & -1.8 & -1.6 & -1.4 & -1.2 & -1 \end{pmatrix} \\ &K = \texttt{place}(A, B, p) = \begin{pmatrix} -1.5456 & -0.3636 & -0.1654 & 0.0854 & 0.0847 \\ 0.0668 & 0.0156 & -0.6457 & -0.677 & -0.6302 \end{pmatrix} \\ &G = I_2 \end{aligned} \tag{23}$$

If we have a stable observer $\dot{\hat{x}} = CLx + (A - LC)\hat{x} + Bu$, applying $u = -K\hat{x}$ will stabilize the system (separation principle). Assuming that $G = I_2$, we chose L, C_1, D_1 such that the observer be stable, and the closed loop system with $y_p = D_1 Cx + C_1\hat{x}$ performance input be strict output passive (from r to y_p). We obtain:

$$\begin{aligned} &C_1 = \begin{pmatrix} 0.0048 & 0.0022 & 0.0015 & -0.001 & -0.0009 \\ -0.0005 & 0.0018 & -0 & 0.0044 & 0.0006 \end{pmatrix}, \; D_1 = \begin{pmatrix} 0.0031 & 0.0001 \\ 0.0083 & -0.0005 \end{pmatrix} \\ &L^T = \begin{pmatrix} -11.8203 & 11.0372 & 2.8894 & -13.6842 & 7.1631 \\ -3.5266 & -2.2015 & 7.9634 & -0.3426 & -0.4627 \end{pmatrix} \end{aligned} \tag{24}$$

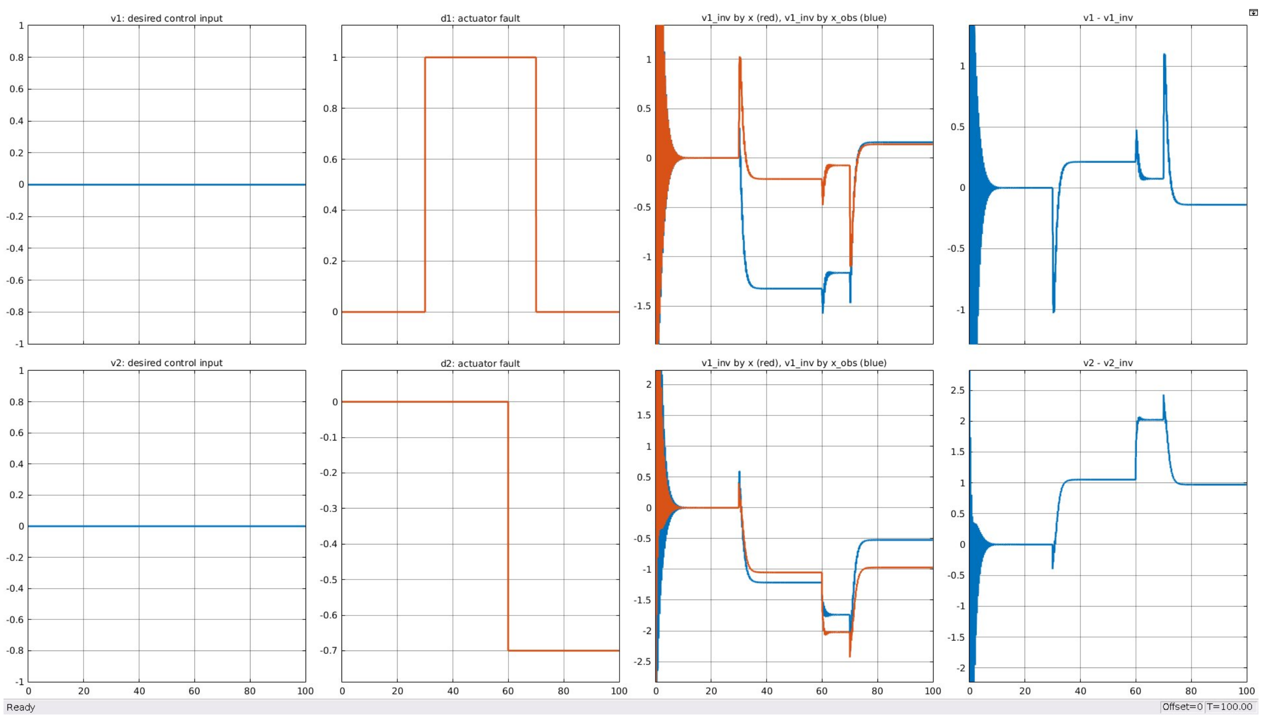
Poles and zeros of the open-loop system:

$$\texttt{POLES} = \begin{pmatrix} -5 & -2 & -1 & -3 & -2 \end{pmatrix}, \; \texttt{ZEROS} = \begin{pmatrix} -3.3395 & -0.3282 & -1.2477 \end{pmatrix} \tag{25}$$



References

[1] A Isidori and AJ Krener. On feedback equivalence of nonlinear systems. *Systems & Control Letters*, 2(2):118–121, 1982.


v1: desired control input
d1: actuator fault
v1_inv by x (red), v1_inv by x_obs (blue)
v1 - v1_inv
v2: desired control input
d2: actuator fault
v1_inv by x (red), v1_inv by x_obs (blue)
v2 - v2_inv
Ready
Offset=0
T=100.00