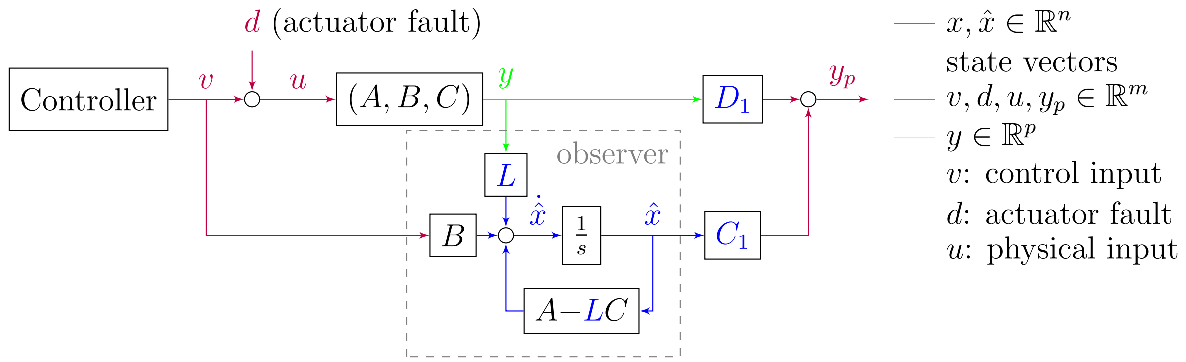
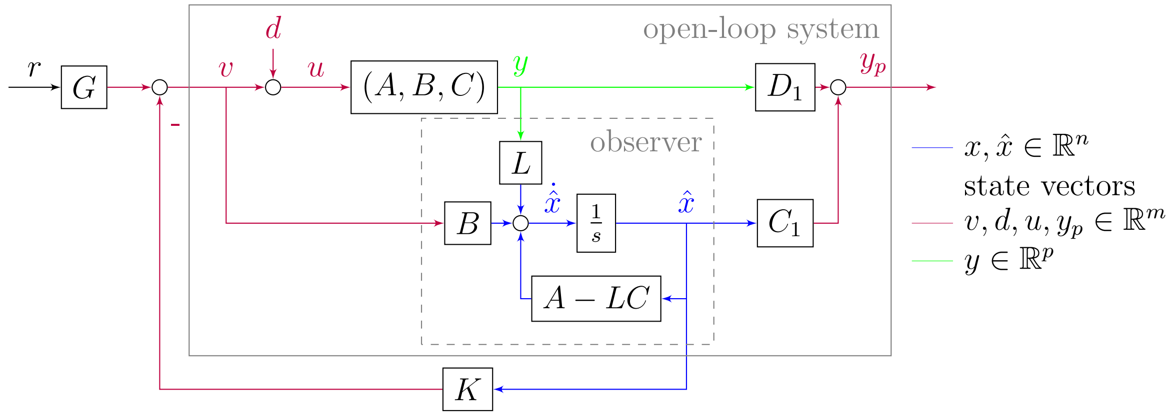
1 MIMO, $u \to y_p$ passivization, feedback equivalence with a passive system

Having an LTI MIMO system, which is somehow fed back through a controller (either tuned by output vector or by the full state vector). The actuator is assumed to be faulty. We intend to detect its fault using system inversion.]. However, the system is not invertible, since its zeros are unstable and/or its vector relative degree (v.r.d.) is more than 1, therefore, we augment the system with an additional (linear) dynamics, which is tuned by the system's output y and the designed control input u, hence its resemblance to an observer.



The goal is to chose matrices L, C_1, D_1 such that the system $v \to y_p$ be feedback equivalent to a passive system and the v.r.d. of $v \to y_p$ be 1.



1.1 Numerical examples – MIMO

Unstable MIMO system:

$$H_1(s) = \begin{pmatrix} \frac{s-1}{(s+1)(s-2)} & \frac{1}{s-3} \\ \frac{(s+2)(s-7)}{(s-5)(s-1)(s-2)} & \frac{s-6}{(s-2)(s-3)} \end{pmatrix} \tag{1}$$

$$\texttt{tzero(H1)} = \begin{pmatrix} 7.2854 & 0.0214 & 3 & 2 & 1.8361 \end{pmatrix}$$

Stable MIMO system:

$$H_2(s) = \begin{pmatrix} \frac{s-1}{(s+1)(s+2)} & \frac{1}{s+3} \\ \frac{s-7}{s^2+6s+5} & \frac{s-6}{s^2+5s+6} \end{pmatrix} \tag{2}$$

$$\texttt{tzero(H2)} = \begin{pmatrix} 2.5-7.1937i & 2.5+7.1937i \end{pmatrix}$$

For both models $CB = \left(\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix}\right)$ is rank deficient ($r \neq \{1,1\}$).

2 Separation principle, $X = \left(\begin{smallmatrix} x \\ \hat{x} \end{smallmatrix}\right)$ $K, L = \texttt{place}$, $G = I_m$, $C_1, D_1 = \texttt{sdpvar}$

$$\text{OLS:} \quad \begin{cases} \dot{x} = Ax + Bu \\ \dot{\hat{x}} = LCx + (A-LC)\hat{x} + Bu \\ y_p = D_1 Cx + C_1 \hat{x} \end{cases} \Rightarrow \begin{cases} \widehat{A}_o = \begin{pmatrix} A & 0 \\ LC & A-LC \end{pmatrix}, \quad \widehat{B}_o = \begin{pmatrix} B \\ B \end{pmatrix} \\ \widehat{C}_o = \begin{pmatrix} D_1 C & C_1 \end{pmatrix} \end{cases} \tag{3}$$

$$\text{CLS:} \quad u = -K\hat{x} + Gu \Rightarrow \begin{cases} \widehat{A} = \begin{pmatrix} A & -BK \\ LC & A-LC-BK \end{pmatrix}, \quad \widehat{B} = \widehat{B}_o G \\ \widehat{C} = \begin{pmatrix} D_1 C & C_1 \end{pmatrix} \end{cases} \tag{4}$$

$$\dot{V}(X) \leq r^T y_p + y_p^T r - y_p^T W y_p, \quad \text{where } W \succ 0,\ V(X) = X^T P X,\ P = P^T \succ 0 \tag{5}$$

$$\begin{pmatrix} X \\ r \end{pmatrix}^T \begin{pmatrix} \widehat{A}^T P + P\widehat{A} & P\widehat{B} \\ \widehat{B}^T P & 0 \end{pmatrix} \begin{pmatrix} X \\ r \end{pmatrix} \leq \begin{pmatrix} X \\ r \end{pmatrix}^T \begin{pmatrix} -\widehat{C}^T W \widehat{C} & \widehat{C}^T \\ \widehat{C} & 0 \end{pmatrix} \begin{pmatrix} X \\ r \end{pmatrix} \tag{6}$$

Equivalently: (it could be **only** negative **semi**-definite)

$$M_0 = \begin{pmatrix} \widehat{A}^T P + P\widehat{A} + \widehat{C}^T W \widehat{C} & P\widehat{B} - \widehat{C}^T \\ \widehat{B}^T P - \widehat{C} & 0 \end{pmatrix} \preceq 0, \quad \text{but} \not\prec 0 \tag{7}$$

Using Schur's complement lemma: (it could be **only** negative **semi**-definite)

$$M_1 = \begin{pmatrix} \widehat{A}^T P + P\widehat{A} & P\widehat{B} - \widehat{C}^T & \widehat{C}^T \\ \widehat{B}^T P - \widehat{C} & 0 & 0 \\ \widehat{C} & 0 & -W^{-1} \end{pmatrix} \preceq 0, \quad \text{but} \not\prec 0 \quad \text{(LMI)} \tag{8}$$

Known variables: A, B, C, D (model matrices), furthermore, G, K, L are preliminarily designed.

Free variables: C_1, D_1 (output matrices), $P = \begin{pmatrix} Q & \\ & S \end{pmatrix}$

2.1 Numerical results (in case of $H_1(s)$ – unstable)

Before optimization:

$$\begin{aligned} p &= \begin{pmatrix} -2 & -1.8 & -1.6 & -1.4 & -1.2 & -1 \end{pmatrix} \\ L^T &= \texttt{place}(A^T, B^T, p)^T = \begin{pmatrix} -9.45 & -82.2 & 206 & 57.3 & -193 & -199 \\ -85.7 & 58.7 & -50.2 & -0.158 & 16 & 16 \end{pmatrix} \\ K &= \texttt{place}(A, B, p) = \begin{pmatrix} 5.34 & -0.398 & 0.323 & -0.888 & 0.443 & -0.445 \\ -5.05 & 3.6 & -3.13 & 1.61 & 3.27 & -0.0894 \end{pmatrix} \\ G &= I_2 \end{aligned} \tag{9}$$

Ezáltal kaptunk egy stabil observert és az observer által megfigyelt állapotot visszacsatolva egy stabil zárt hurkot.

Ezek után válasszuk meg C_1 és D_1-et úgy, hogy a zárt rendszer passzív is legyen, tehát az open-loop rendszer feedback ekvivalens egy passzív rendszerrel és az $r = \{1, \ldots, 1\}$ by construction teljesül, ezért az open-loop rendszer minimum fázisú.

Az optimalizációs eljárás után kapjuk, hogy:

$$C_1 = \begin{pmatrix} 2.81 & -0.758 & -7.06 & 2.13 & 5.98 & -12.3 \\ -1.39 & -2.21 & -0.601 & -1.98 & 6.4 & -6.2 \end{pmatrix}, \quad D_1 = \begin{pmatrix} -7.18 & -6.53 \\ -10.3 & -0.0594 \end{pmatrix} \tag{10}$$

Poles and zeros of the open-loop system:

$$\texttt{POLES} = \begin{pmatrix} 5 & -1 & 2 & 1 & 3 & 2 \end{pmatrix}, \quad \texttt{ZEROS} = \begin{pmatrix} -0.31+1.14i & -0.31-1.14i & -0.35 & -0.22 \end{pmatrix} \tag{11}$$

3 Separation principle (advanced, linearised), $X = \left(\begin{smallmatrix} x \\ e \end{smallmatrix}\right)$ $K = \texttt{place}$, $G = I_m$, $C_1, D_1 = \texttt{sdpvar}$, $P := \left(\begin{smallmatrix} Q & 0 \\ 0 & S \end{smallmatrix}\right)$, $N := SL$

$$\text{OLS:} \quad \begin{cases} \dot{x} = Ax + Bu \\ \dot{\hat{x}} = LCx + (A-LC)\hat{x} + Bu \\ \dot{e} = \dot{x} - \dot{\hat{x}} = (A-LC)e \\ y_p = D_1 Cx + C_1\hat{x} \\ \quad = (D_1 C + C_1)x - C_1 x + C_1 \hat{x} \\ \quad = (D_1 C + C_1)x - C_1 e \end{cases} \Rightarrow \begin{cases} \widetilde{A}_o = \begin{pmatrix} A & 0 \\ 0 & A-LC \end{pmatrix}, \quad \widetilde{B}_o = \begin{pmatrix} B \\ 0 \end{pmatrix} \\ \widetilde{C}_o = \begin{pmatrix} D_1 C + C_1 & -C_1 \end{pmatrix} \end{cases} \tag{12}$$

$$\text{CLS:} \quad \begin{aligned} u &= -K\hat{x} + Gu \\ &= -Kx + Kx - K\hat{x} + Gu \\ &= -Kx + Ke + Gu \end{aligned} \Rightarrow \begin{cases} \widetilde{A} = \begin{pmatrix} A-BK & BK \\ 0 & A-LC \end{pmatrix}, \quad \widetilde{B} = \widetilde{B}_o G \\ \widetilde{C} = \begin{pmatrix} D_1 C + C_1 & -C_1 \end{pmatrix} \end{cases} \tag{13}$$

Using Schur's complement lemma: (it could be **only** negative **semi**-definite)

$$M_1 = \begin{pmatrix} \widetilde{A}^T P + P\widetilde{A} & P\widetilde{B} - \widetilde{C}^T & \widetilde{C}^T \\ \widetilde{B}^T P - \widetilde{C} & 0 & 0 \\ \widetilde{C} & 0 & -W^{-1} \end{pmatrix} \preceq 0, \quad \text{but} \not\prec 0 \quad \text{(will be an LMI)} \tag{14}$$

If we constrain $P = \begin{pmatrix} Q & \\ & S \end{pmatrix}$ to be a *block-diagonal* symmetric positive definite matrix, than

$$\widetilde{A}^T P + P\widetilde{A} = \begin{pmatrix} Q(A-BK) + (A-BK)^T Q & QBK \\ K^T B^T Q & SA + A^T S - SLC - C^T L^T S \end{pmatrix} \tag{15}$$

We can introduce $N := SL$ (not overdetermined), obtaining a linear matrix variable:

$$\widetilde{A}^T P + P\widetilde{A} = \begin{pmatrix} Q(A-BK) + (A-BK)^T Q & QBK \\ K^T B^T Q & SA + A^T S - NC - C^T N^T \end{pmatrix} \tag{16}$$

Known variables: A, B, C, D (model matrices), furthermore, K, G are preliminarily designed.

Free variables: C_1, D_1, L (model matrices of the augmented model), $P = \begin{pmatrix} Q & \\ & S \end{pmatrix}$

3.1 Numerical results (in case of $H_2(s)$ – stable)

We design a stabilizing feedback gain K for the original system ($u = -Kx$):

$$\begin{aligned} p &= \begin{pmatrix} -2 & -1.8 & -1.6 & -1.4 & -1.2 & -1 \end{pmatrix} \\ K &= \texttt{place}(A, B, p) = \begin{pmatrix} -1.5456 & -0.3636 & -0.1654 & 0.0854 & 0.0847 \\ 0.0668 & 0.0156 & -0.6457 & -0.677 & -0.6302 \end{pmatrix} \\ G &= I_2 \end{aligned} \tag{17}$$

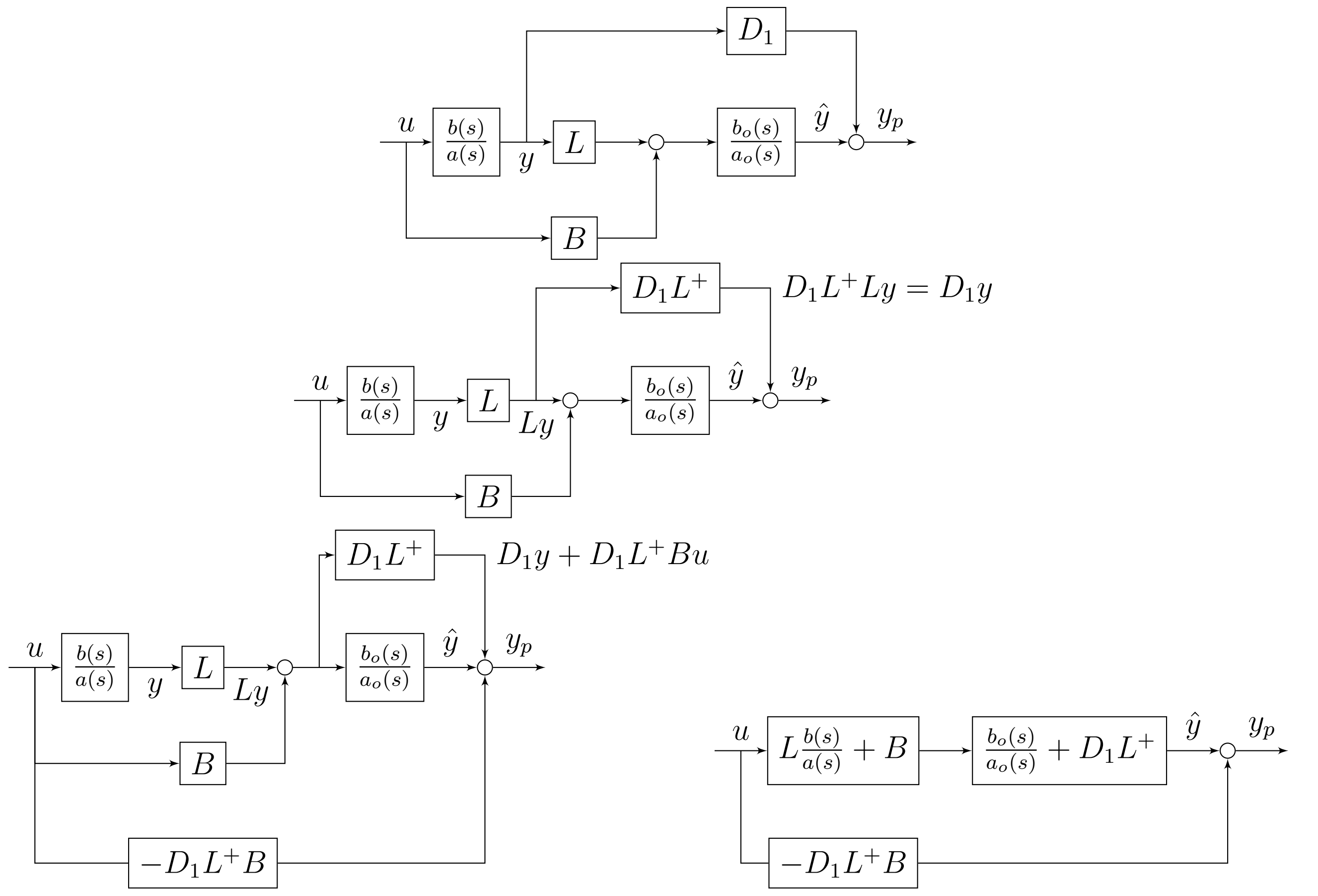
If we have a stable observer $\dot{\hat{x}} = CLx + (A-LC)\hat{x} + Bu$, applying $u = -K\hat{x}$ will stabilize the system (separation principle). Assuming that $G = I_2$, we chose L, C_1, D_1 such that the observer be stable, and the closed loop system with $y_p = D_1 Cx + C_1\hat{x}$ performance input be strict output passive (from r to y_p). We obtain:

$$\begin{aligned} C_1 &= \begin{pmatrix} 0.0048 & 0.0022 & 0.0015 & -0.001 & -0.0009 \\ -0.0005 & 0.0018 & -0 & 0.0044 & 0.0006 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 0.0031 & 0.0001 \\ 0.0083 & -0.0005 \end{pmatrix} \\ L^T &= \begin{pmatrix} -11.8203 & 11.0372 & 2.8894 & -13.6842 & 7.1631 \\ -3.5266 & -2.2015 & 7.9634 & -0.3426 & -0.4627 \end{pmatrix} \end{aligned} \tag{18}$$

Poles and zeros of the open-loop system:

$$\texttt{POLES} = \begin{pmatrix} -5 & -2 & -1 & -3 & -2 \end{pmatrix}, \quad \texttt{ZEROS} = \begin{pmatrix} -3.3395 & -0.3282 & -1.2477 \end{pmatrix} \tag{19}$$

4 Megvalósíthatósági tanulmány – operátortartománybeli analízis



Ahol

$$\frac{b_o(s)}{a_o(s)} = C_1\big(sI - A + LC\big)^{-1} \tag{20}$$

Szerintem ez MIMO esetben is jó ($a(s)$ és $a_o(s)$ skalár, minden egyéb mátrix):

$$\begin{aligned}
y_p &= D_1y + \hat{y} = \frac{1}{a(s)}D_1b(s)u + \frac{1}{a_o(s)}b_o(s)\left(\frac{1}{a(s)}Lb(s) + B\right)u \\
G_e(s) &= \frac{1}{a(s)}D_1b(s) + \frac{1}{a_o(s)}b_o(s)\left(\frac{1}{a(s)}Lb(s) + B\right) \\
G_e(s) &= \frac{1}{a(s)a_o(s)}D_1b(s)a_o(s) + \frac{1}{a(s)a_o(s)}b_o(s)(Lb(s) + Ba(s)) \\
G_e(s) &= \frac{1}{a(s)a_o(s)}\Big(D_1b(s)a_o(s) + b_o(s)Lb(s) + b_o(s)Ba(s)\Big)
\end{aligned} \tag{21}$$

Tehát az új számláló:

$$b_e(s) = D_1b(s)a_o(s) + b_o(s)Lb(s) + b_o(s)Ba(s) \tag{22}$$

A piros tagok lehetnek instabilak. Kék: megválasztható szabad constans. Mivel $a_o(s)$ skalár:

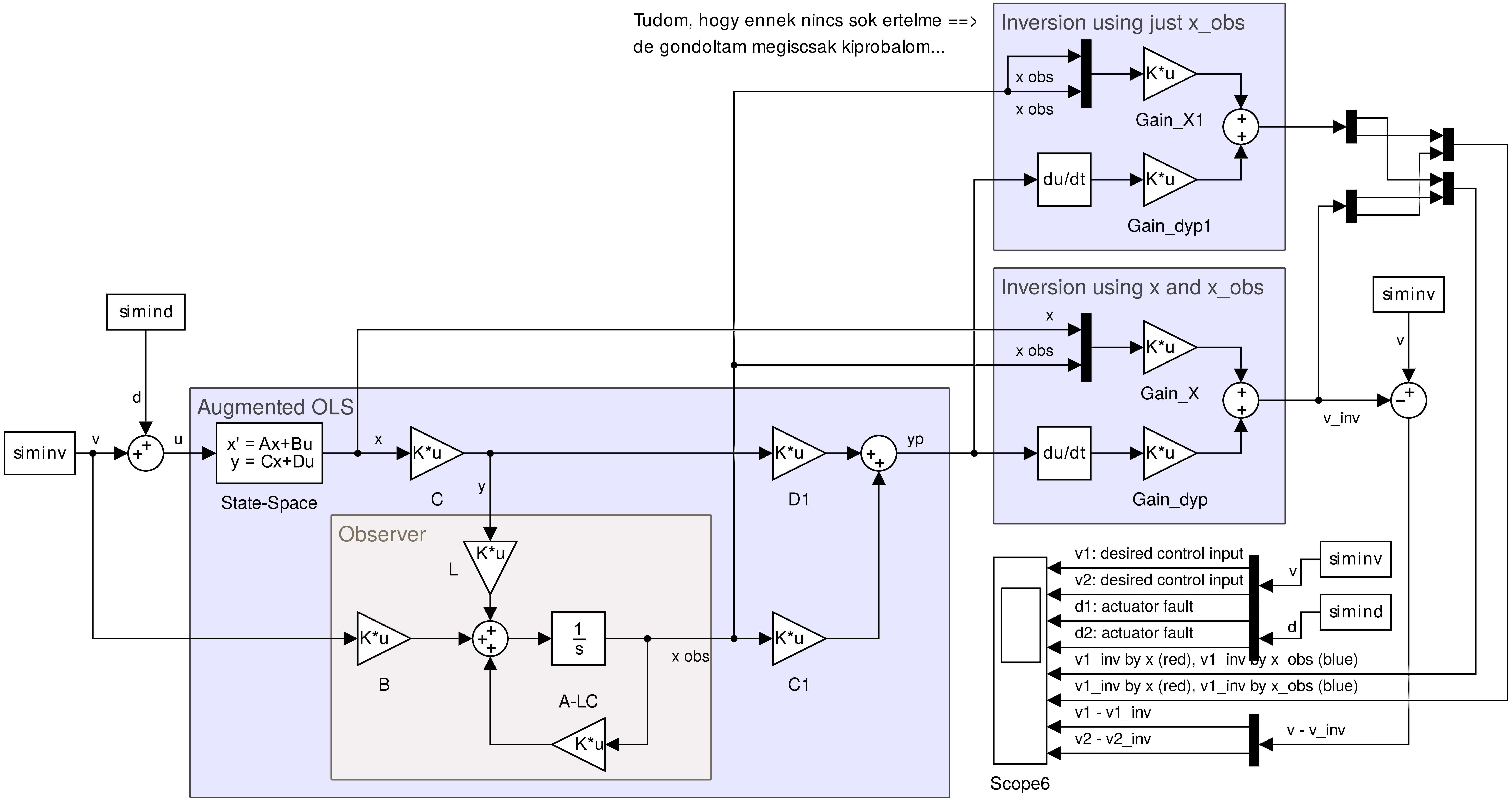
$$b_e(s) = \Big(a_o(s)D_1 + b_o(s)L\Big)b(s) + b_o(s)Ba(s) \leftarrow \text{ legyen stabil} \tag{23}$$

Numerikus példával illusztrálom, hogy ez valóban lehetséges:

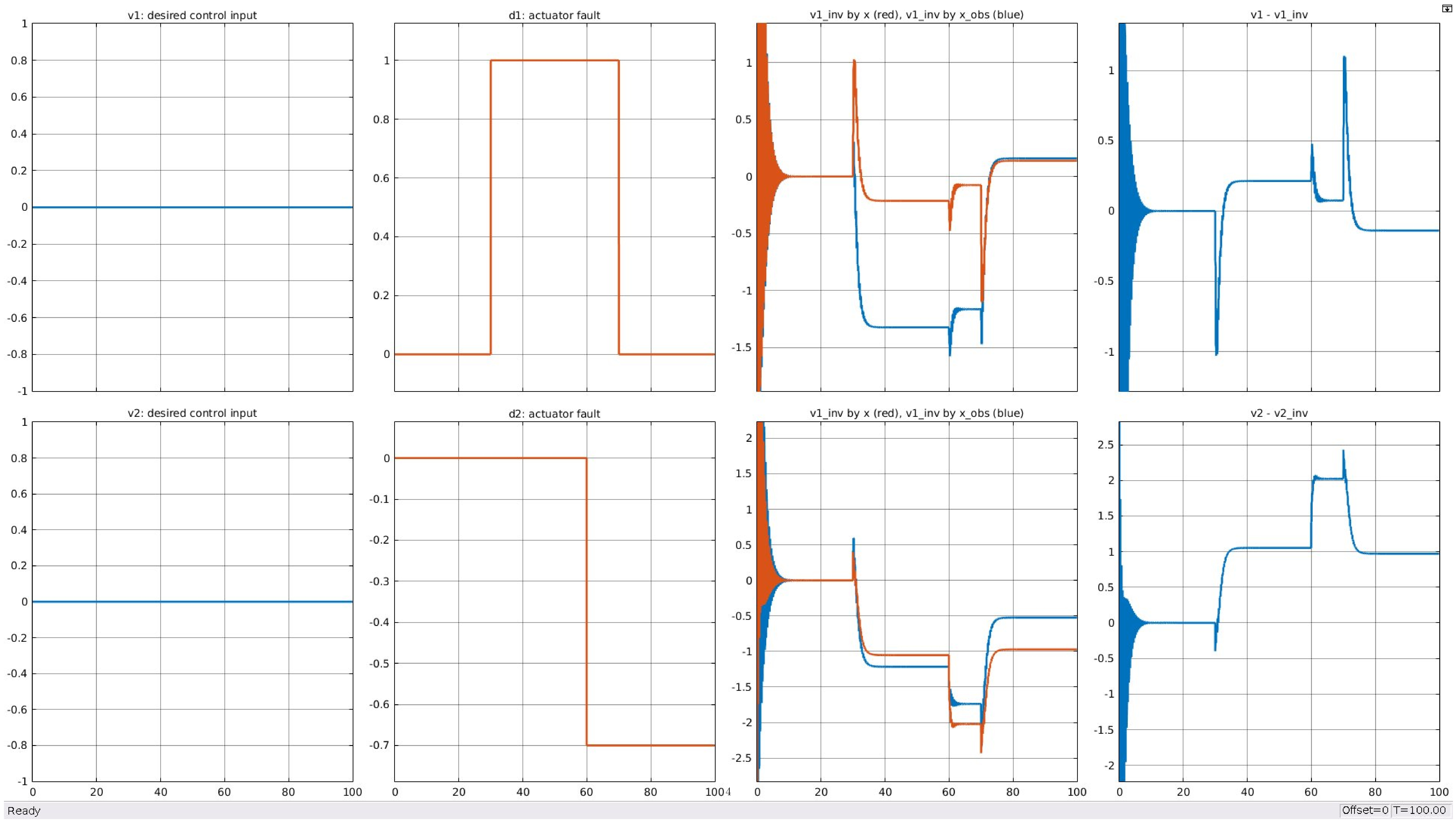
$$\begin{aligned}
&G(s) = \frac{b(s)}{a(s)} = \frac{s-5}{s^2-3s+2}, \quad A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \; C = \begin{pmatrix} 0.5 & -2.5 \end{pmatrix} \\
&\text{observer: } L \overset{\texttt{place}}{:=} \begin{pmatrix} -13 \\ -5 \end{pmatrix}, \; C_1 := \begin{pmatrix} 1 & -1 \end{pmatrix} \Rightarrow G_o(s) = \frac{b_o(s)}{a_o(s)} = \frac{1}{s^2+3s+2}\begin{pmatrix} s+9 & -s-25 \end{pmatrix} \\
&b_e(s) = (D_1+2)s^3 + (4-2D_1)s^2 + (-13D_1-2)s - (10D_1+4)\Big|_{D_1=-1} \\
&\quad\; = s^3 + 6s^2 + 11s + 6 = (s+3)(s+2)(s+1) \\
&\text{eredő átviteli függvény: } G_e(s) = \frac{s+3}{s^2-3s+2}
\end{aligned} \tag{24}$$

Másik példa (ami a relatív degreet is megjavítja):

$$\begin{aligned}
&G(s) = \frac{s-5}{(s-3)(s^2-3s+2)}, \quad L^T \overset{\texttt{place}}{:=} \begin{pmatrix} -193 & -212 & -52 \end{pmatrix}, \; C_1 := \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}, \; D_1 := 0.1 \\
&G_e(s) = \frac{s^2+4.1s+3.5}{s^3-6s^2+11s-6}
\end{aligned} \tag{25}$$



Tudom, hogy ennek nincs sok ertelme ==>
de gondoltam megiscsak kiprobalom...
Inversion using just x_obs
x obs
x obs
K*u
Gain_X1
du/dt
K*u
Gain_dyp1
Inversion using x and x_obs
x
x obs
K*u
Gain_X
du/dt
K*u
Gain_dyp
siminv
v
v_inv
simind
d
siminv
v
u
x' = Ax+Bu
y = Cx+Du
State-Space
Augmented OLS
x
K*u
C
y
K*u
D1
yp
Observer
K*u
L
K*u
B
1
s
A-LC
K*u
x obs
K*u
C1
v1: desired control input
v2: desired control input
d1: actuator fault
d2: actuator fault
v1_inv by x (red), v1_inv by x_obs (blue)
v1_inv by x (red), v1_inv by x_obs (blue)
v1 - v1_inv
v2 - v2_inv
siminv
v
simind
d
v - v_inv
Scope6


v1: desired control input
d1: actuator fault
v1_inv by x (red), v1_inv by x_obs (blue)
v1 - v1_inv
v2: desired control input
d2: actuator fault
v1_inv by x (red), v1_inv by x_obs (blue)
v2 - v2_inv
Ready
Offset=0
T=100.00