6.057
Introduction to MATLAB

Orhan Celiker, IAP 2019

Course Layout

Problem sets

- One per day, should take about 4 hours to complete
- Submit Word or PDF, include code and figures
- Some questions optional, but highly recommended!

Requirements for passing

- Attend 3/4 lectures (Friday is optional)
- Complete all problem sets (graded on a 3-level scale: -, √, +)...
- ... and achieve √ average

Prerequisites: You'll be fine!

MATLAB Basics

- MATLAB can be thought of as a super-powerful graphing calculator
  - Remember the TI-83 from calculus?
  - With many more buttons (built-in functions)

- In addition, it is a programming language
  - MATLAB is an interpreted language, like Python
  - Commands are executed line-by-line

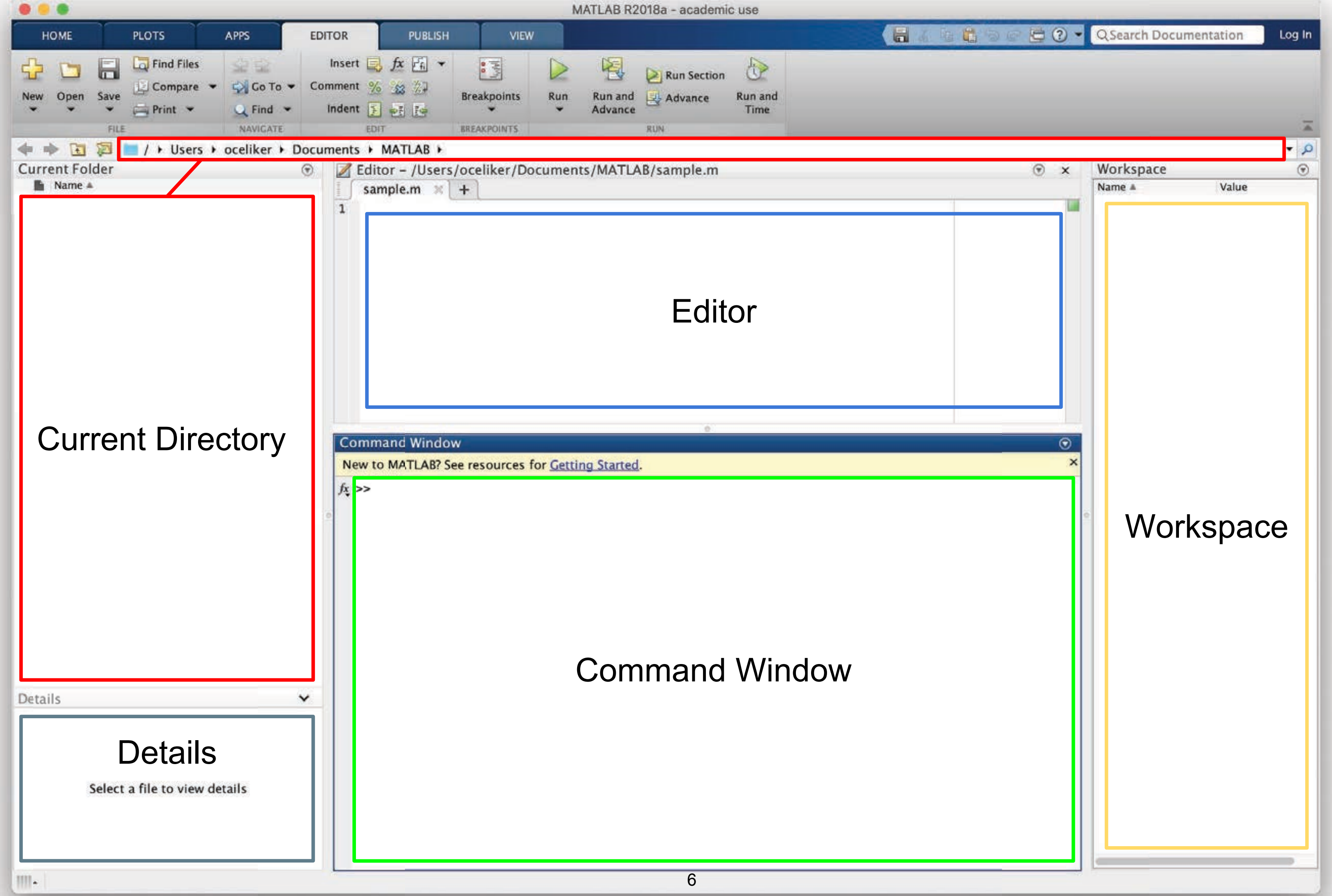
Outline

I. <u>Getting Started</u>
II. Scripts
III. Making Variables
IV. Manipulating Variables
V. Basic Plotting

Getting Started

- To get MATLAB Student Version for yourself

- You can also use MATLAB online
    - https://matlab.mathworks.com (requires Mathworks account with license)


MATLAB R2018a - academic use
HOME
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BREAKPOINTS
Run
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/ Users oceliker Documents MATLAB
Current Folder
Name
Current Directory
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Details
Select a file to view details
Editor – /Users/oceliker/Documents/MATLAB/sample.m
sample.m
Editor
Command Window
New to MATLAB? See resources for Getting Started.
Command Window
Workspace
Name
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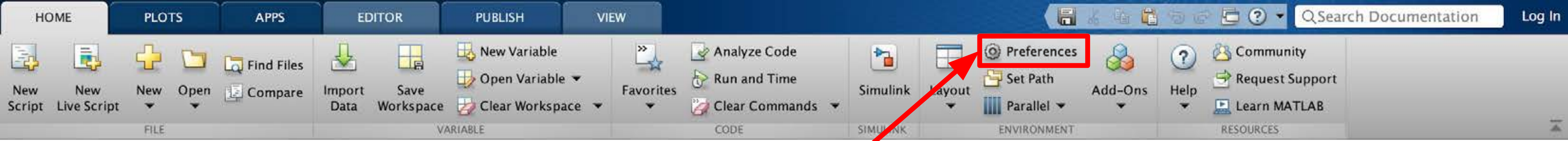
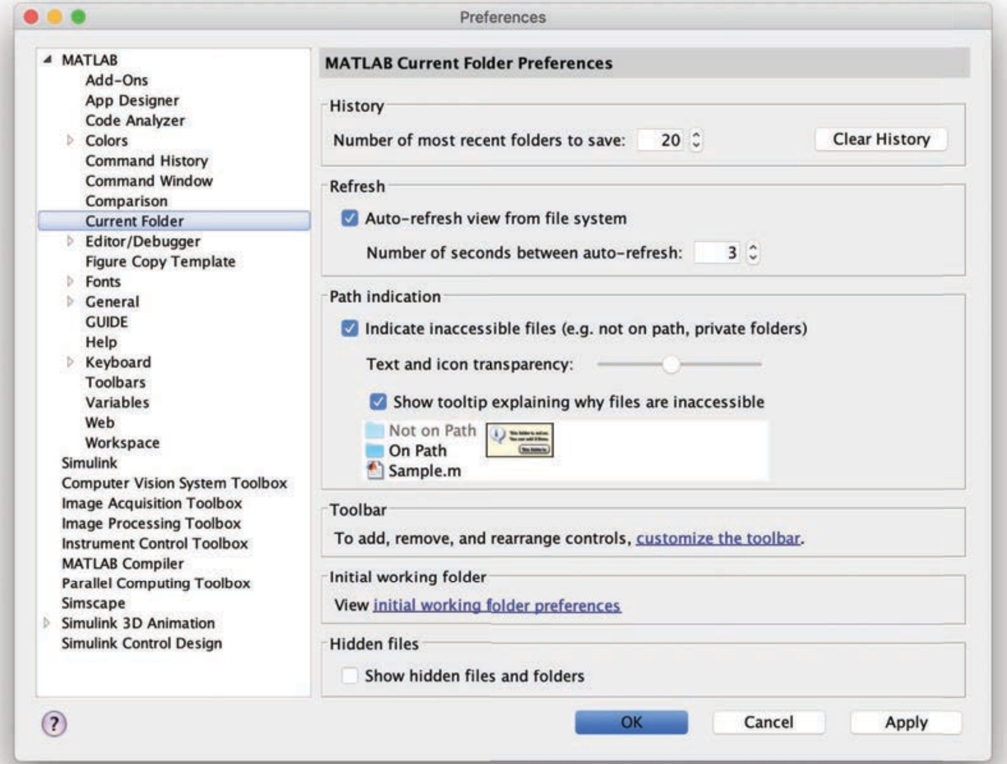
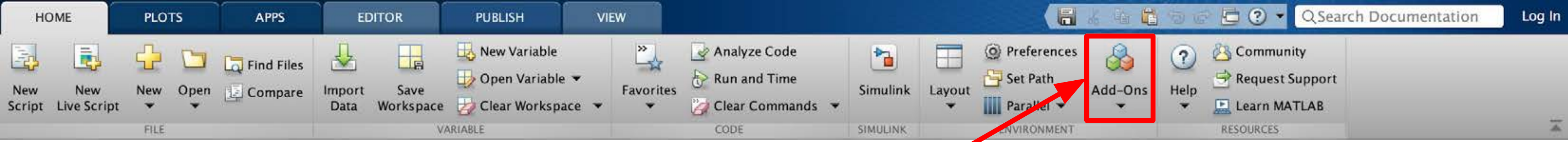


Customization

- In the top ribbon, navigate to:
  Home -> Environment -> Preferences

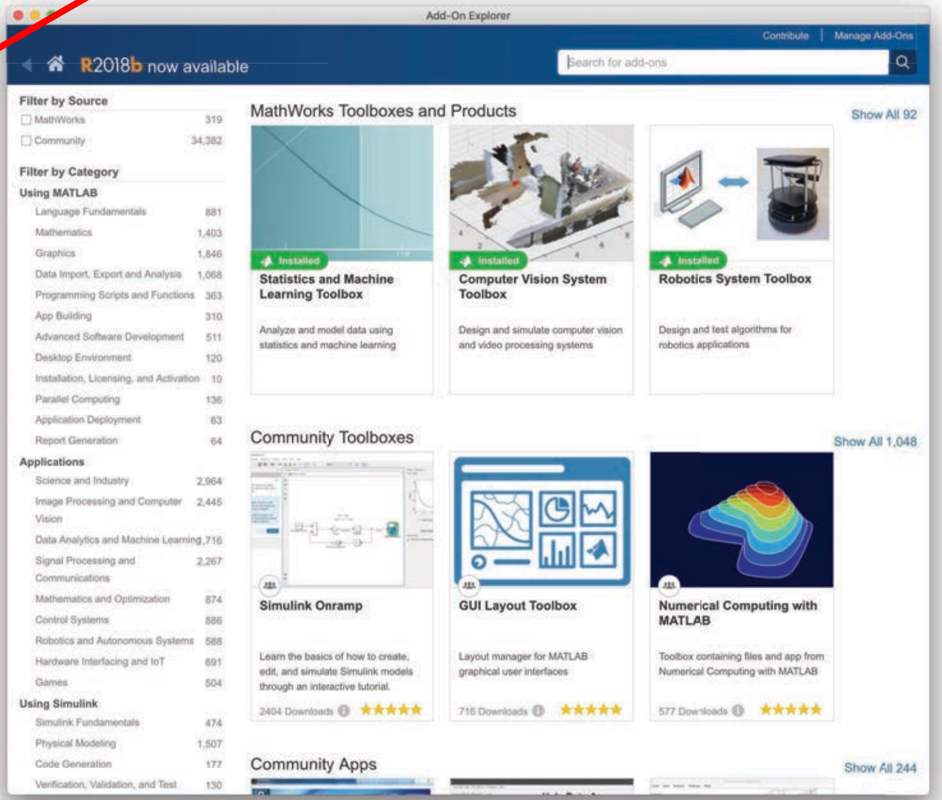
- Allows you to customize your MATLAB experience (colors, fonts, etc.)





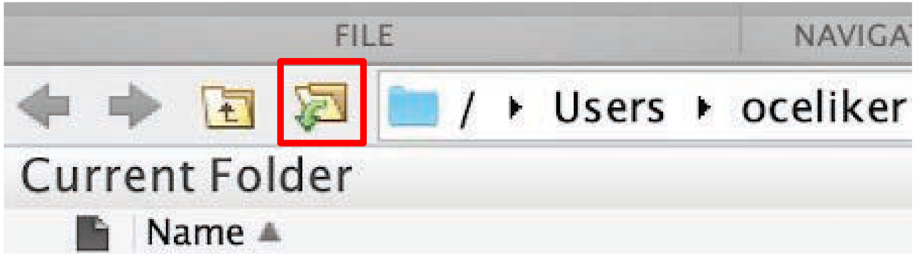
Installing Toolboxes

- In the top ribbon, navigate to:
  Home -> Environment -> Add-Ons
- Allows you to install toolboxes included with your license
- Recommended toolboxes:
  - Curve Fitting Toolbox
  - Computer Vision System Toolbox
  - Image Processing Toolbox
  - Optimization Toolbox
  - Signal Processing Toolbox
  - and anything related to your field!



Making Folders

- Use folders to keep your programs organized
- To make a new folder, click "Browse" next to the file path



- Click the Make New Folder button, and change the name of the folder. In the MATLAB folder (which should be open by default), make the following folder structure:

  MATLAB
  ↳ IAP MATLAB
  ↳ Day1

Help/Docs

- `help`
  - The most important command for learning MATLAB on your own!
- To get info on how to use a function:
  - `help sin`
    - Help lists related functions at the bottom and links to the documentation
- To get a nicer version of help with examples and easy-to-read description:
  - `doc sin`
- To search for a function by specifying keywords:
  - `docsearch sin trigonometric`

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Scripts: Overview

- Scripts are
  - Collection of commands executed in sequence
  - Written in the MATLAB editor
  - Saved as m-files (.m extension)
- To create an m-file from the command line:
  - `edit MyFileName.m`
  - or click the "New Script" button on the top left

Scripts: Some notes

- COMMENT!
  - Anything following a % sign is interpreted as a comment
  - The first contiguous comment becomes the script's help file
  - Comment thoroughly to avoid wasting time later!
  - Mark beginning of a code block by using %%
- Note that scripts are somewhat static, with no explicit input and output
- All variables created or modified in a script retain their values after script execution

Exercise: Scripts

- Make a script with the name **helloWorld.m**
- When run, the script should show the following text:

```
Hello world!
I am going to learn MATLAB!
```

Hint: Use `disp(...)` to display strings. Strings are written between single quotes, e.g. `'This is a string'`

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Variable Types

- MATLAB is a "weakly typed" language
  - No need to initialize variables!
- MATLAB supports various types; the most popular ones are
  - 3.84
    - 64-bit double (default)
  - 'A'
    - 16-bit char
- Most variables you'll deal with are vectors, matrices, doubles or chars
- Other types are also supported: complex, symbolic, 16-bit and 8-bit integers (uint16 & uint8), etc.

Naming Variables

- To create a variable, simply assign a value to a name:

```
myNumberVariable = 3.14
myStringVariable = 'hello world!'
```

- Variable name rules
  - First character must be a LETTER
  - After that, any combination of numbers, letters and _
  - Names are CASE-SENSITIVE (e.g. `var1` is different than `Var1`)

Naming Variables (cont.)

Built-in variables (don't use these names for anything else!):

`i, j:` can be used to indicate complex numbers*

`pi:` has the value 3.1415...

`ans:` stores the result of the last unassigned value

`Inf, -Inf:` infinities

`NaN:` "Not a Number"

ops, use `ii, jj, kk`, etc. for loop counters.

Scalars

- A variable can be given a value explicitly
  - `a = 10`
  - Shows up in workspace!
- Or as a function of explicit values and existing variables
  - `c = 1.3 * 45 - 2 * a`
- To suppress output, end the line with a semicolon
  - `cooldude = 13/3;`

Arrays

- Like other programming languages, arrays are an important part of MATLAB
- Two types of arrays:
  - Matrix of numbers (either double or complex)
  - Cell array of objects (more advanced data structure)

MATLAB makes vectors easy!
That’s its power!

Row vectors

- Row vector: comma- or space-separated values between square brackets
    - `row = [ 1 2 3.2 4 6 5.4 ];`
    - `row = [ 1, 2, 4, 7, 4.3, 1.1 ];`
- Command window:

```
>> row=[1 2 5.4 -6.6]

row =

    1.0000    2.0000    5.4000   -6.6000
```

- Workspace:

Workspace

Stack: Base

Name	Size	Bytes	Class
row	1x4	32	double array

Column vectors

- Column vector: semicolon-separated values between square brackets
  - `col = [ 1; 2; 3.2; 4; 6; 5.4 ];`

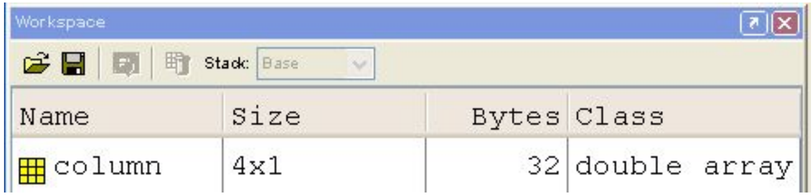
- Command window:

```
>> column=[4;2;7;4]

column =

     4
     2
     7
     4
```

- Workspace:



Size and length

- You can tell the difference between a row and a column by:
  - Looking in the workspace
  - Displaying the variable in the command window
  - Using the size function

```
>> size(row)

ans =

     1     4

>> length(row)

ans =

     4
```

```
>> size(column)

ans =

     4     1

>> length(column)

ans =

     4
```

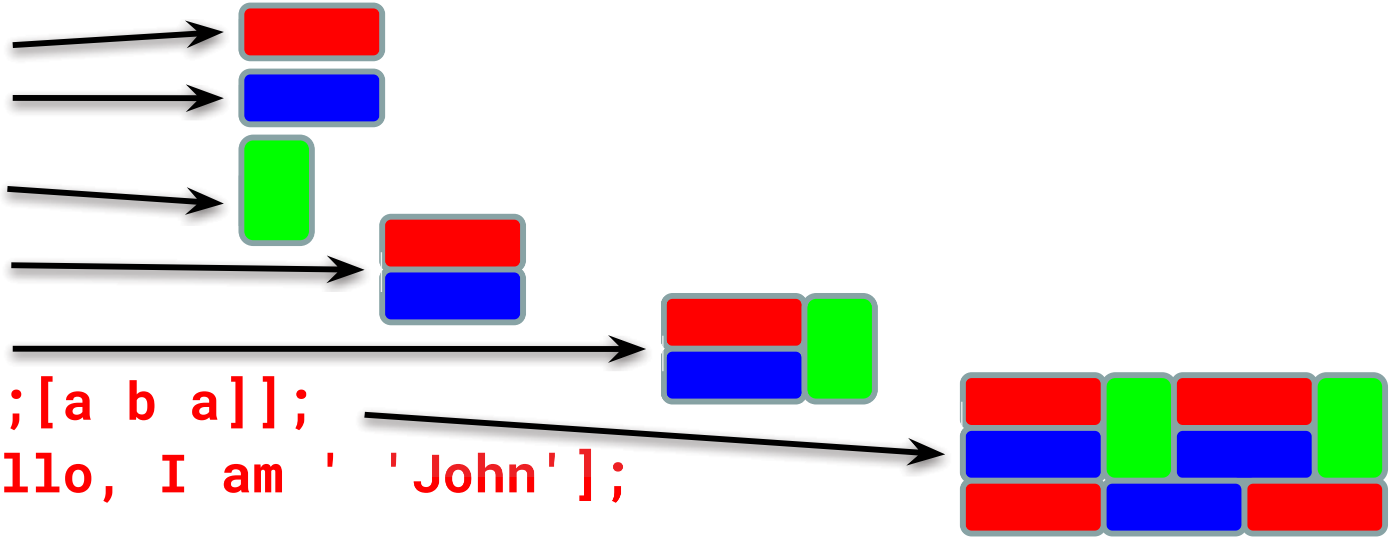
Matrices

- Make matrices like vectors
  - Element by element
    - `a= [1 2;3 4];`

$$a = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

- By concatenating vectors or matrices (dimension matters)

```
a = [1 2];
b = [3 4];
c = [5;6];
d = [a;b];
e = [d c];
f = [[e e];[a b a]];
str = ['Hello, I am ' 'John'];
```



- Strings are character vectors

save/clear/load

- Use save to save variables to a file
  - `save myFile a b`
  - Saves variables a and b to the file myFile.mat in the current directory
  - Default working directory is MATLAB unless you navigate to another folder
  - Make sure you are in the correct folder. Right now we should be in \MATLAB\IAP MATLAB\Day 1
- Use clear to save variables to a file
  - `clear a b`
  - Look at workspace: variables a and b are gone
- Use load to load variables into the workspace
  - `load myFile`
  - Look at workspace: a and b are back

Exercise: Variables

Get and save the current date and time

- Create a variable `start` using the function `clock`
- What is the size of `start`? Is it a row or column?
- What does `start` contain? See `help clock`
- Convert the vector start to a string. Use the function `datestr` and name the new variable `startString`
- Save `start` and `startString` into a mat file named `startTime`

Exercise: Variables II

- In helloWorld.m, read in variables you saved using `load`
- Display the following text:

  I started learning MATLAB on [date, time]

- Hint: Use the `disp` command again
- Remember that strings are just vectors of characters, so you can join two strings by making a row vector with the two strings as sub-vectors.

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Basic Scalar Operations

- Arithmetic operations (+, -, *, /)
  - `7/45`
  - `(1+1i)*(1+2i)`
  - `1/0`
  - `0/0`
- Exponentiation
  - `4^2`
  - `(3+4*1j)^2`
- Complicated expressions: use parentheses
  - `((2+3)*3)^0.1`

Built-in Functions

- MATLAB has an <u>enormous</u> library of built-in functions
- Call using parentheses, passing parameters to function
  - `sqrt(2)`
  - `log(2), log10(0.23)`
  - `cos(1.2), atan(-.8)`
  - `exp(2+4*1i)`
  - `round(1.4), floor(3.3), ceil(4.23)`
  - `angle(1i); abs(1+1i);`

Exercise: Scalars

helloWorld script:

- Your learning time constant is 1.5 days. Calculate the number of seconds in 1.5 days and name this variable tau
- This class lasts 5 days. Calculate the number of seconds in 5 days and name this variable endOfClass
- This equation describes your knowledge as a function of time t:

$$k = 1 - e^{-t/\tau}$$

- How well will you know MATLAB at endOfClass? Name this variable knowledgeAtEnd (use exp)
- Using the value of knowledgeAtEnd, display the phrase:

```
At the end of 6.057, I will know X% of MATLAB
```

Hint: to convert a number to a string, use num2str

Transpose

- The transpose operator turns a column vector into a row vector, and vice versa
  - `a = [1 2 3 4+i]`
  - `transpose(a)`
  - `a'`
  - `a.'`
- The ' gives the Hermitian-transpose
  - Transposes and conjugates all complex numbers
- For vectors of real numbers .' and ' give same result
  - For transposing a vector, always use .' to be safe

Addition and Subtraction

- Addition and subtraction are element-wise
- Sizes must match (unless one is a scalar):

$$\begin{array}{r} [12 \quad 3 \quad 32 \quad -11] \\ +[2 \quad 11 \quad -30 \quad 32] \\ \hline =[14 \quad 14 \quad 2 \quad 21] \end{array}$$

$$\begin{bmatrix} 12 \\ 1 \\ -10 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ -1 \\ 13 \\ 33 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ -23 \\ -33 \end{bmatrix}$$

Addition and Subtraction

- `c = row + column`

Use the transpose to make sizes compatible

- `c = row.' + column`
- `c = row + column.'`

Can sum up or multiply elements of vector

- `s=sum(row);`
- `p=prod(row);`

Element-wise functions

- All the functions that work on scalars also work on vectors
  - ```
    t = [1 2 3];
    f = exp(t);
    ```
    is the same as
    ```
    f = [exp(1) exp(2) exp(3)];
    ```
- If in doubt, check a function's help file to see if it handles vectors element-wise
- Operators (* / ^) have two modes of operation
  - element-wise
  - standard
    ```

Element-wise functions

- To do element-wise operations, use the dot: . (.*, ./, .^)
- BOTH dimensions must match (unless one is scalar)!

```
a=[1 2 3];b=[4;2;1];

a.*b  , a./b  , a.^b → all errors

a.*b.', a./b.’, a.^(b.’) → all valid
```

Operators

- Multiplication can be done in a standard way or element-wise
- Standard multiplication (*) is matrix product
  - Remember from linear algebra: inner dimensions must MATCH!!
- Standard exponentiation (^) can only be done on square matrices or scalars
- Left and right division (/ \) is same as multiplying by inverse
  - Our recommendation: for now, just multiply by inverse (more on this later)

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} * \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} = 11$$

$$1 \times 3 * 3 \times 1 = 1 \times 1$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \wedge 2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Must be square to do powers

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 6 & 12 & 18 \\ 9 & 18 & 27 \end{bmatrix}$$

$$3 \times 3 * 3 \times 3 = 3 \times 3$$

Exercise: Vector Operations

Calculate how many seconds elapsed since start of class

- In helloWorld.m, make variables called secPerMin, secPerHour, secPerDay, secPerMonth (assume 30.5 days per month), and secPerYear (12 months in year), which have the number of seconds in each time period
- Assemble a row vector called secondConversion that has elements in this order: secPerYear, secPerMonth, secPerDay, secPerHour, secPerMin, 1
- Make a currentTime vector by using clock
- Compute elapsedTime by subtracting currentTime from start
- Compute t (the elapsed time in seconds) by taking the dot product of secondConversion and elapsedTime (transpose one of them to get the dimensions right)

Exercise: Vector Operations

Display the current state of your knowledge

- Calculate currentKnowledge using the same relationship as before, and the t we just calculated:

$$k = 1 - e^{-t/\tau}$$

- Display the following text:
  At this time, I know X% of MATLAB

Automatic Initialization

- Initialize a vector of **ones**, **zeros**, or **random** numbers
  - » `o=ones(1,10)`
    - ➢ Row vector with 10 elements, all 1
  - » `z=zeros(23,1)`
    - ➢ Column vector with 23 elements, all 0
  - » `r=rand(1,45)`
    - ➢ Row vector with 45 elements (uniform (0,1))
  - » `n=nan(1,69)`
    - ➢ Row vector of NaNs (representing uninitialized variables)

Automatic Initialization

- To initialize a linear vector of values use **linspace**
  - » **`a=linspace(0,10,5)`**
    - ➢ Starts at 0, ends at 10 (inclusive), 5 values
- Can also use colon operator (**:**)
  - » **`b=0:2:10`**
    - ➢ Starts at 0, increments by 2, and ends at or before 10
    - ➢ Increment can be decimal or negative
  - » **`c=1:5`**
    - ➢ If increment is not specified, default is 1
- To initialize logarithmically spaced values use **logspace**
    - ➢ Similar to **linspace**, but see **help**

Exercise: Vector Functions

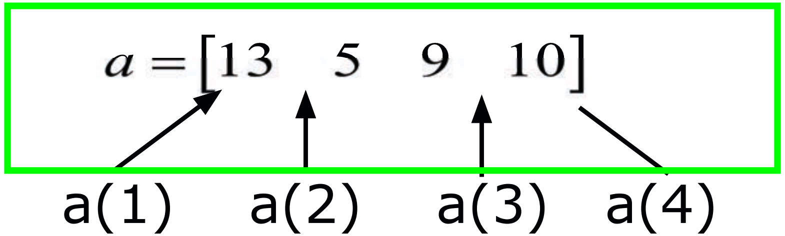
Calculate your learning trajectory

- In helloWorld.m, make a linear time vector `tVec` that has 10,000 samples between 0 and `endOfClass`
- Calculate the value of your knowledge (call it `knowledgeVec`) at each of these time points using the same equation as before:

$$k = 1 - e^{-t/\tau}$$

Vector Indexing

- <u>MATLAB indexing starts with **1**, not **0**</u>
  - ➢ We will not respond to any emails where this is the problem.
- a(n) returns the nth element

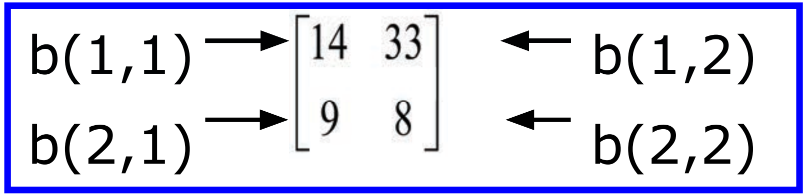


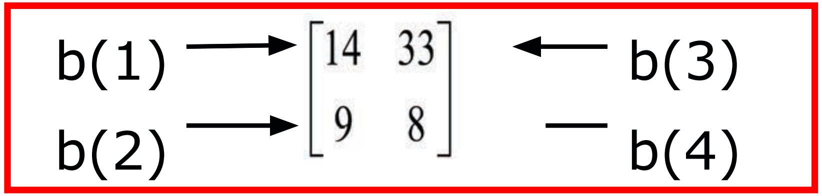
- The index argument can be a vector. In this case, each element is looked up individually, and returned as a vector of the same size as the index vector.

```
» x=[12 13 5 8];
```

Matrix Indexing

- Matrices can be indexed in two ways
    - ➢ using **subscripts** (row and column)
    - ➢ using linear **indices** (as if matrix is a vector)
- Matrix indexing: subscripts or linear indices





- Picking submatrices


```
» A = rand(5) % shorthand for 5x5 matrix
```

Advanced Indexing 1

- To select rows or columns of a matrix, use the **:**

$$c = \begin{bmatrix} 12 & 5 \\ -2 & 13 \end{bmatrix}$$

```
» d=c(1,:);          d=[12 5];
» e=c(:,2);          e=[5;13];
» c(2,:)=[3 6];   %replaces second row of c
```

Advanced Indexing 2

- MATLAB contains functions to help you find desired values
  - » `vec = [5 3 1 9 7]`
- To get the minimum value and its index (similar for `max`):
  - » `[minVal,minInd] = min(vec);`
- To find the indices of specific values or ranges
  - » `ind = find(vec == 9); vec(ind) = 8;`
  - » `ind = find(vec > 2 & vec < 6);`
    - ➢ **find** expressions can be very complex, more on this later
    - ➢ When possible, **logical indexing** is faster than **find**!
    - ➢ E.g., `vec(vec == 9) = 8;`

Exercise: Indexing

When will you know 50% of MATLAB?

- First, find the index where **knowledgeVec** is closest to 0.5. Mathematically, what you want is the index where the value of ~ $\left|knowledgeVec - 0.5\right|$ is at a minimum (use **abs** and **min**)
- Next, use that index to look up the corresponding time in **tVec** and name this time **halfTime**
- Finally, display the string: Convert **halfTime** to days by using secPerDay. I will know half of MATLAB after X days

Outline

(1) Getting Started
(2) Scripts
(3) Making Variables
(4) Manipulating Variables
(5) **Basic Plotting**

Did everyone sign in?

Plotting

- Example

```
» x=linspace(0,4*pi,10);
» y=sin(x);
```

- Plot values against their index

```
» plot(y);
```

- Usually we want to plot y versus x

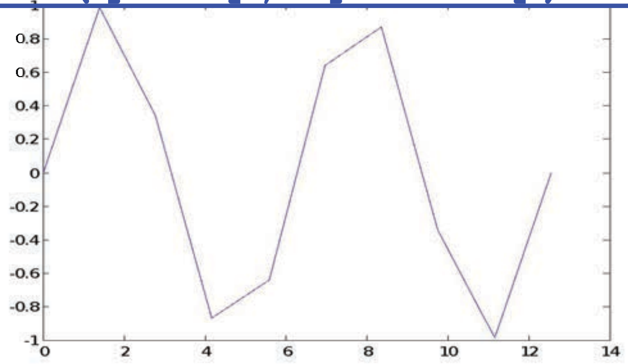
```
» plot(x,y);
```

MATLAB makes visualizing data fun and easy!

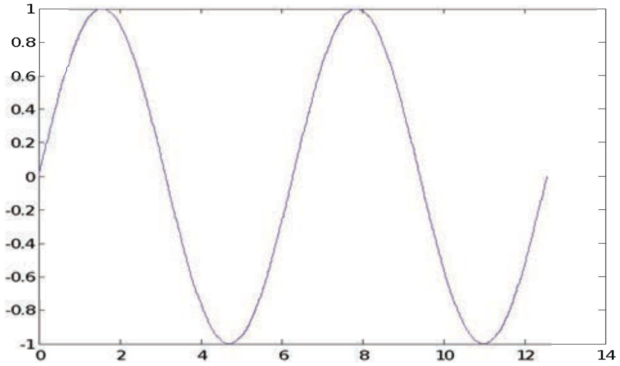
What does plot do?

- **plot** generates dots at each (x,y) pair and then connects the dots with a line
- To make plot of a function look smoother, evaluate at more points
  - » `x=linspace(0,4*pi,1000);`
  - » `plot(x,sin(x));`
- x and y vectors must be same size or else you'll get an error
  - » `plot([1 2], [1 2 3])`

10 x values:



1000 x values:



Exercise: Plotting

Plot the learning trajectory

- In helloWorld.m, open a new figure (use `figure`)
- Plot knowledge trajectory using `tVec` and `knowledgeVec`
- When plotting, convert `tVec` to days by using `secPerDay`
- Zoom in on the plot to verify that `halfTime` was calculated correctly

End of Lecture 1

(1) Getting Started
(2) Scripts
(3) Making Variables
(4) Manipulating Variables
(5)

Hope that wasn't too much and you enjoyed it!!

6.057
Introduction to programming in MATLAB

Lecture 2: Visualization and Programming

Orhan Celiker

IAP 2019

Homework 1 Recap

Some things that came up:

- Plotting a straight line
  - » `x = 1:10`
  - » `plot(x, 0)`
    - ➢ Not an error, but probably not what you meant

- Use of semicolon – never required if one command per line. You can also put multiple commands on one line; in this case, a semicolon is necessary to separate commands:
  - » `x=1:10; y=(x-5).^2; z = x.*y;`

Plotting

- Example

```
» x=linspace(0,4*pi,10);
» y=sin(x);
```

- Plot values against their index

```
» plot(y);
```

- Usually we want to plot y versus x

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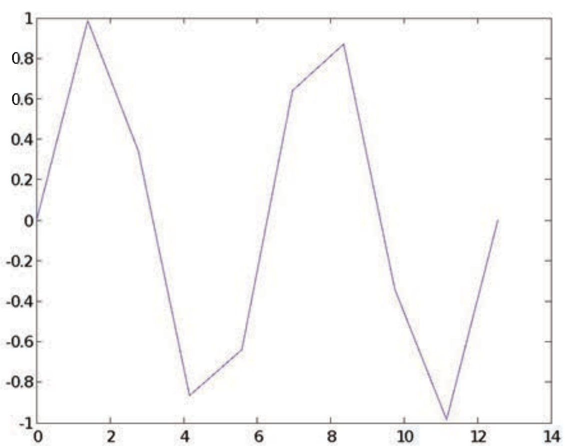
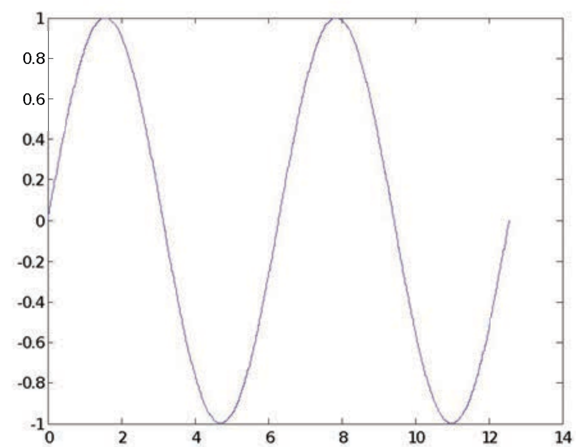
- **plot** generates dots at each (x,y) pair
  and then connects the dots with a line
- To make plot of a function look smoother, evaluate at more points
  » `x=linspace(0,4*pi,1000);`
  » `plot(x,sin(x));`
- x and y vectors must be same size or else you'll get an error
  » `plot([1 2], [1 2 3])`
    ➢ error!!

10 x values:



1000 x values:



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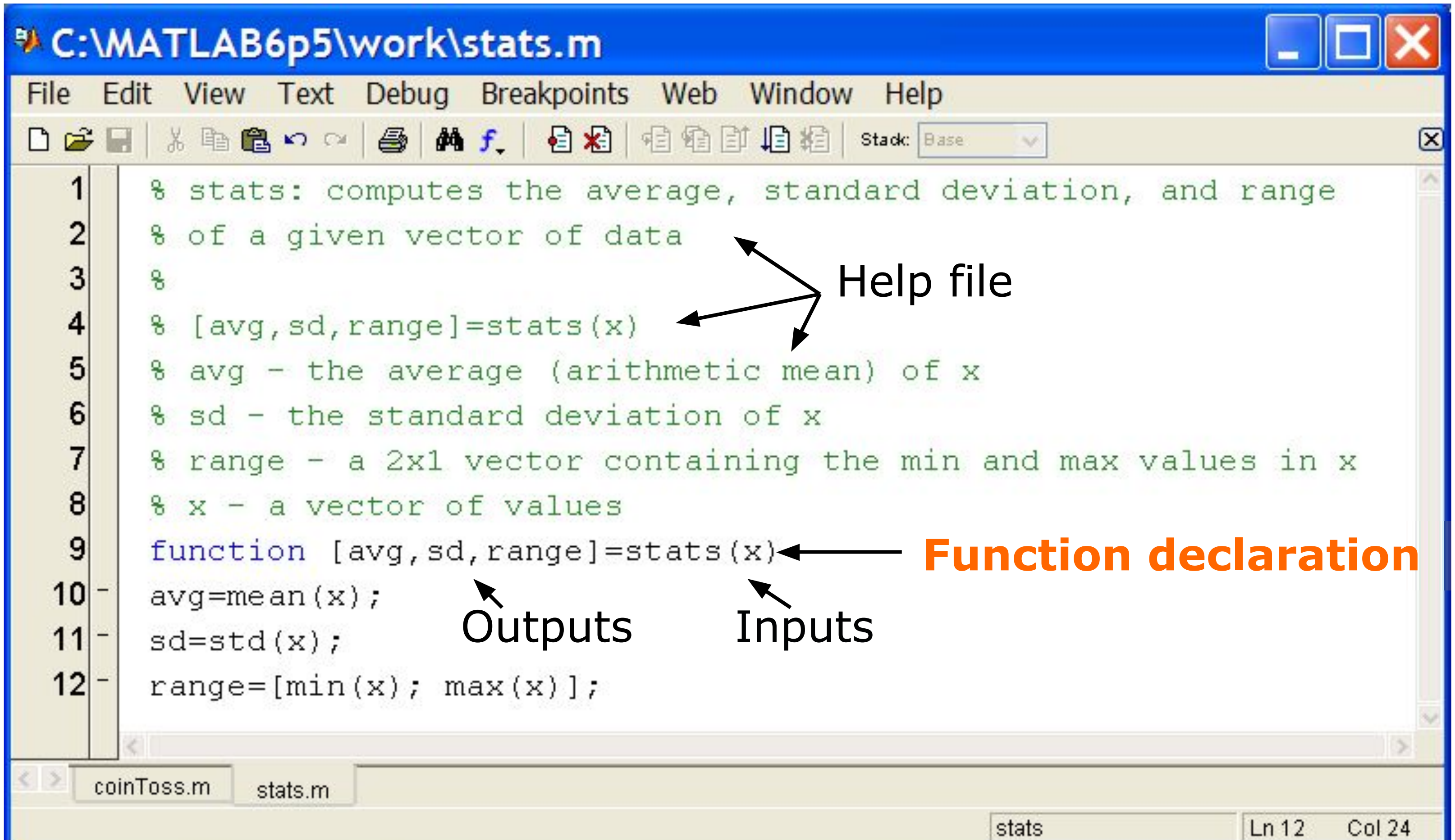
- Zoom in on the plot to verify that `halfTime` was calculated correctly

Outline for Lec 2

(1) **Functions**
(2) Flow Control
(3) Line Plots
(4) Image/Surface Plots
(5) Efficient Codes
(6) Debugging

User-defined Functions

- Functions look exactly like scripts, but for **ONE** difference
  - ➢ Functions must have a function declaration



User-defined Functions

- Some comments about the function declaration

Inputs

```
function [x, y, z] = funName(in1, in2)
```

Must have the reserved word: function

Function name should match m-file name

If more than one output, must be in brackets

- No need for return: MATLAB 'returns' the variables whose names match those in the function declaration (though, you can use **`return`** to break and go back to invoking function)
- Variable scope: Any variable created within the function but not returned disappears after the function stops running (They're called "local variables")

Functions: overloading

- We're familiar with
  - » `zeros`
  - » `size`
  - » `length`
  - » `sum`

- Look at the help file for size by typing
  - » `help size`

- The help file describes several ways to invoke the function
  - ➢ D = SIZE(X)
  - ➢ [M,N] = SIZE(X)
  - ➢ [M1,M2,M3,...,MN] = SIZE(X)
  - ➢ M = SIZE(X,DIM)

Functions: overloading

- MATLAB functions are generally overloaded
  - ➢ Can take a variable number of inputs
  - ➢ Can return a variable number of outputs

- What would the following commands return:

```
» a=zeros(2,4,8); %n-dimensional matrices are OK
» D=size(a)
» [m,n]=size(a)
» [x,y,z]=size(a)
» m2=size(a,2)
```

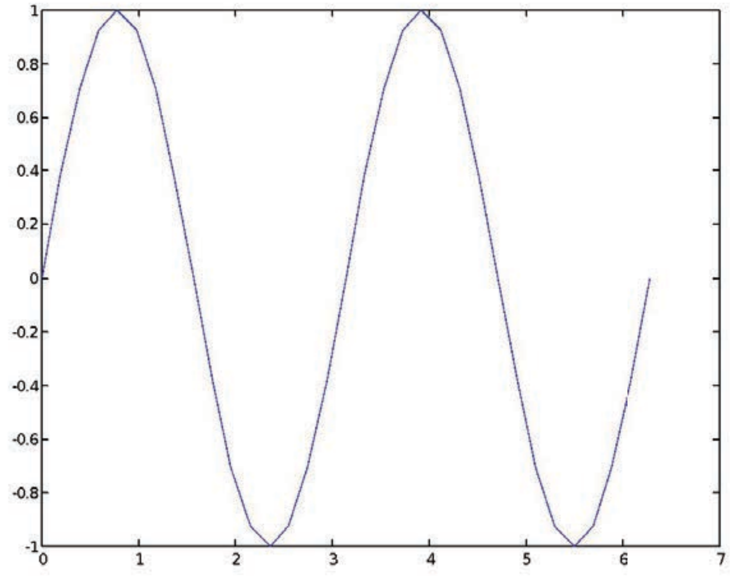
- You can overload your own functions by having variable number of input and output arguments (see `varargin`, `nargin`, `varargout`, `nargout`)

Functions: Exercise

- Write a function with the following declaration:
  `function plotSin(f1)`

- In the function, plot a sine wave with frequency f1, on the interval [0,2π]: $\sin\left(f_1 x\right)$

- To get good sampling, use 16 points per period.



Outline

(1) Functions

(2) Flow Control

(3) Line Plots

(4) Image/Surface Plots

(5) Efficient Codes

(6) Debugging

Relational Operators

- MATLAB uses *mostly* standard relational operators
  - equal ==
  - **not** equal ~=
  - greater than >
  - less than <
  - greater or equal >=
  - less or equal <=
- Logical operators elementwise short-circuit (scalars)
  - And & &&
  - Or | ||
  - **Not** ~
  - Xor xor
  - All true all
  - Any true any

- Boolean values: zero is false, nonzero is true
- See **help .** for a detailed list of operators

if/else/elseif

- Basic flow-control, common to all languages
- MATLAB syntax is somewhat unique

IF

```
if cond
    commands
end
```

Conditional statement: evaluates to true or false

ELSE

```
if cond
    commands1
else
    commands2
end
```

ELSEIF

```
if cond1
    commands1
elseif cond2
    commands2
else
    commands3
end
```

- No need for parentheses: command blocks are between reserved words
- Lots of `elseif`'s? consider using `switch`

for

- **for** loops: use for a known number of iterations
- MATLAB syntax:

Loop variable

```
for n=1:100
    commands
end
```

Command block

- The loop variable
  - ➢ Is defined as a vector
  - ➢ Is a scalar within the command block
  - ➢ Does not have to have consecutive values (but it's usually cleaner if they're consecutive)
- The command block
  - ➢ Anything between the **for** line and the **end**

while

- The while is like a more general for loop:
  - No need to know number of iterations

WHILE

```
while cond
    commands
end
```

- The command block will execute while the conditional expression is true
- Beware of infinite loops! CTRL+C?!
- You can use `break` to exit a loop

Exercise: Conditionals

- Modify your `plotSin(f1)` function to take two inputs: `plotSin(f1,f2)`

- If the number of input arguments is 1, execute the plot command you wrote before. Otherwise, display the line `'Two inputs were given'`
- Hint: the number of input arguments is stored in the built-in variable `nargin`

Outline

(1) **Functions**
(2) **Flow Control**
(3) **Line Plots**
(4) **Image/Surface Plots**
(5) **Efficient Codes**
(6) **Debugging**

Plot Options

- Can change the line color, marker style, and line style by adding a string argument

```
» plot(x,y,'k.-');
```

color → k, marker → ., line-style → -

- Can plot without connecting the dots by omitting line style argument

```
» plot(x,y,'.')
```

- Look at **help plot** for a full list of colors, markers, and line styles

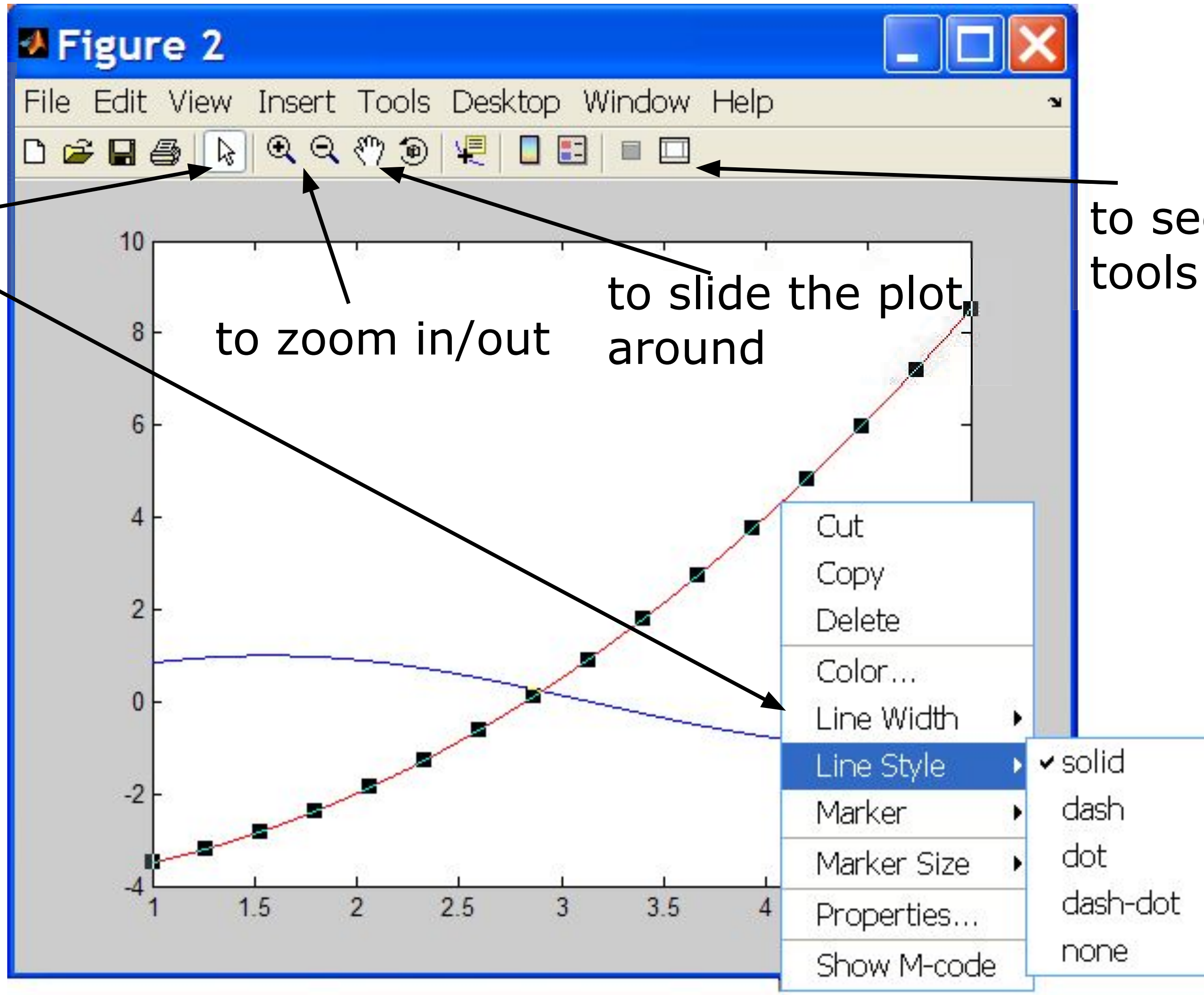
Playing with the Plot

to select lines and delete or change properties

to zoom in/out

to slide the plot around

to see all plot tools at once

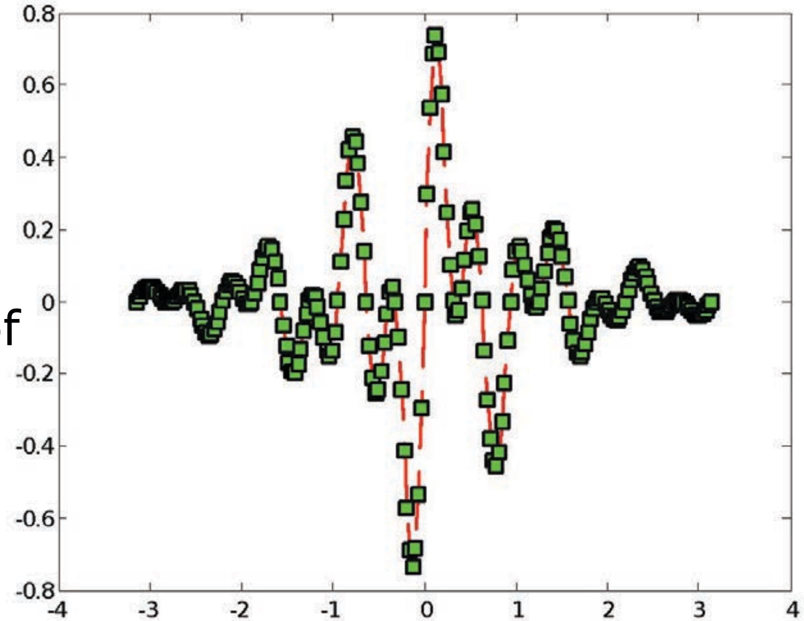


Line and Marker Options

- Everything on a line can be customized

```
» plot(x,y,'s--','LineWidth',2,...
          'Color', [1 0 0], ...
          'MarkerEdgeColor','k',...
          'MarkerFaceColor','g',...
          'MarkerSize',10)
```

You can set colors by using a vector of [R G B] values or a predefined color character like 'g', 'k', etc.



- See **doc line_props** for a full list of properties that can be specified

Cartesian Plots

- We have already seen the plot function

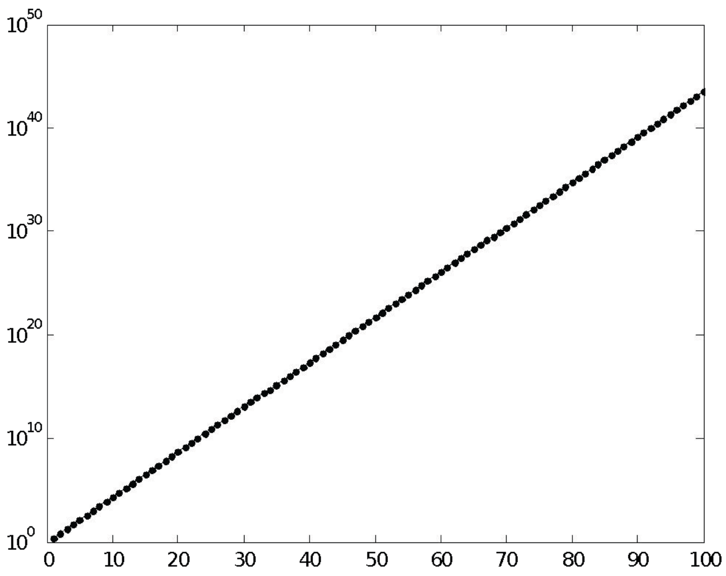
```
» x=-pi:pi/100:pi;
» y=cos(4*x).*sin(10*x).*exp(-abs(x));
» plot(x,y,'k-');
```

- The same syntax applies for semilog and loglog plots

```
» semilogx(x,y,'k');
» semilogy(y,'r.-');
» loglog(x,y);
```

- For example:

```
» x=0:100;
» semilogy(x,exp(x),'k.-');
```



3D Line Plots

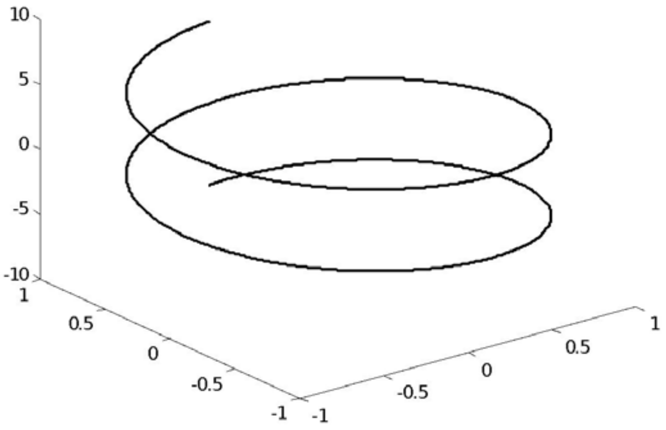
- We can plot in 3 dimensions just as easily as in 2D

```
» time=0:0.001:4*pi;
» x=sin(time);
» y=cos(time);
» z=time;
» plot3(x,y,z,'k','LineWidth',2);
» zlabel('Time');
```

3D Line Plots

- We can plot in 3 dimensions just as easily as in 2D
  - » `time=0:0.001:4*pi;`
  - » `x=sin(time);`
  - » `y=cos(time);`
  - » `z=time;`
  - » `plot3(x,y,z,'k','LineWidth',2);`
  - » `zlabel('Time');`

- Use tools on figure to rotate it
- Can set limits on all 3 axes
  - » `xlim, ylim, zlim`



Axis Modes

- Built-in axis modes (see `doc axis` for more modes)
  - » `axis square`
    - ➢ makes the current axis look like a square box
  - » `axis tight`
    - ➢ fits axes to data
  - » `axis equal`
    - ➢ makes x and y scales the same
  - » `axis xy`
    - ➢ puts the origin in the lower left corner (default for plots)
  - » `axis ij`
    - ➢ puts the origin in the upper left corner (default for matrices/images)

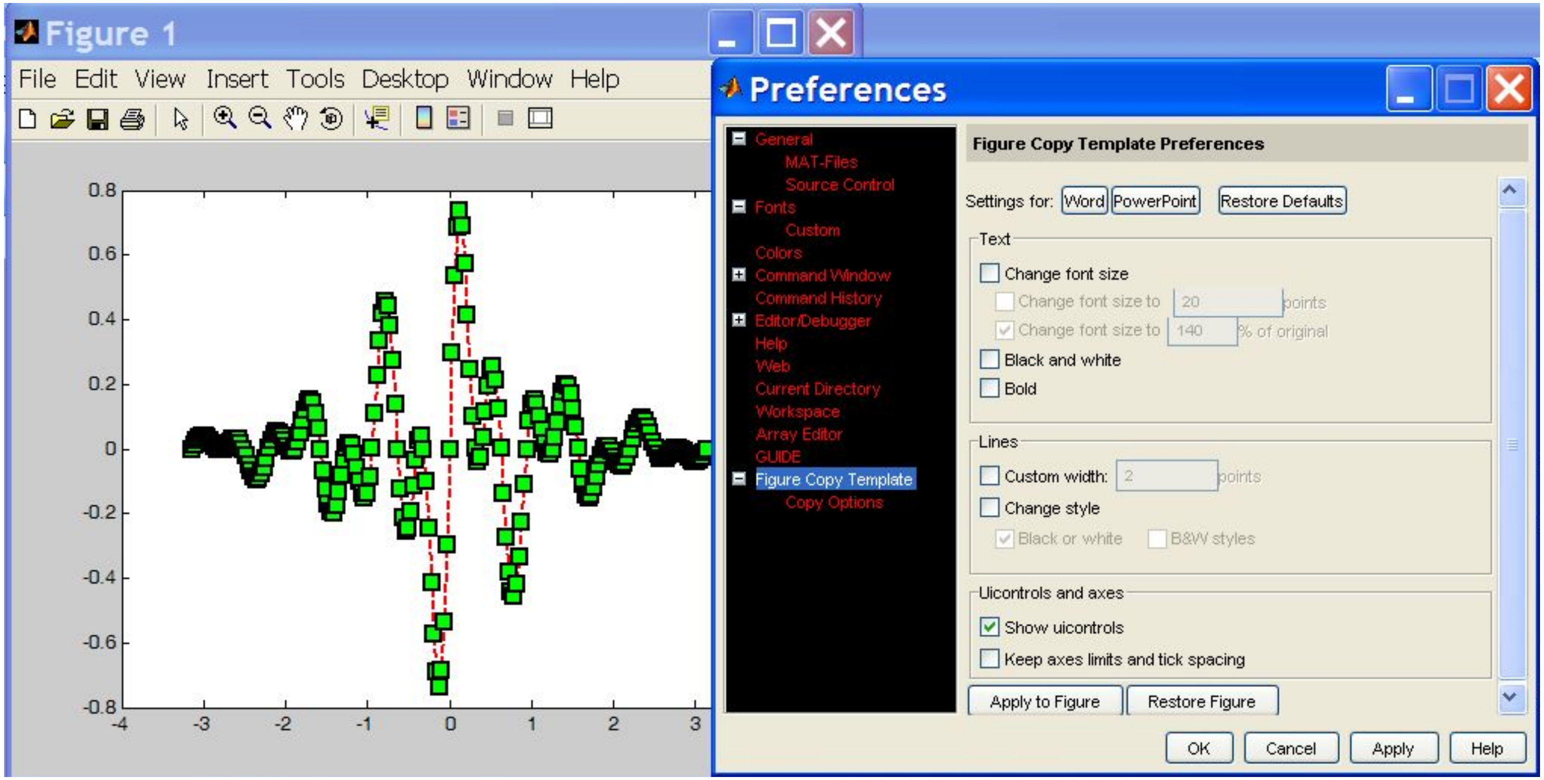
Multiple Plots in one Figure

- To have multiple axes in one figure
  - » `subplot(2,3,1)`
    - ➢ makes a figure with 2 rows and 3 columns of axes, and activates the first axis for plotting
    - ➢ each axis can have labels, a legend, and a title
  - » `subplot(2,3,4:6)`
    - ➢ activates a range of axes and fuses them into one

- To close existing figures
  - » `close([1 3])`
    - ➢ closes figures 1 and 3
  - » `close all`
    - ➢ closes all figures (useful in scripts)

Copy/Paste Figures

- Figures can be pasted into other apps (word, ppt, etc)
- *Edit→ copy options→ figure copy template*
  - ➢ Change font sizes, line properties; presets for word and ppt
- *Edit→ copy figure* to copy figure
- Paste into document of interest



Saving Figures

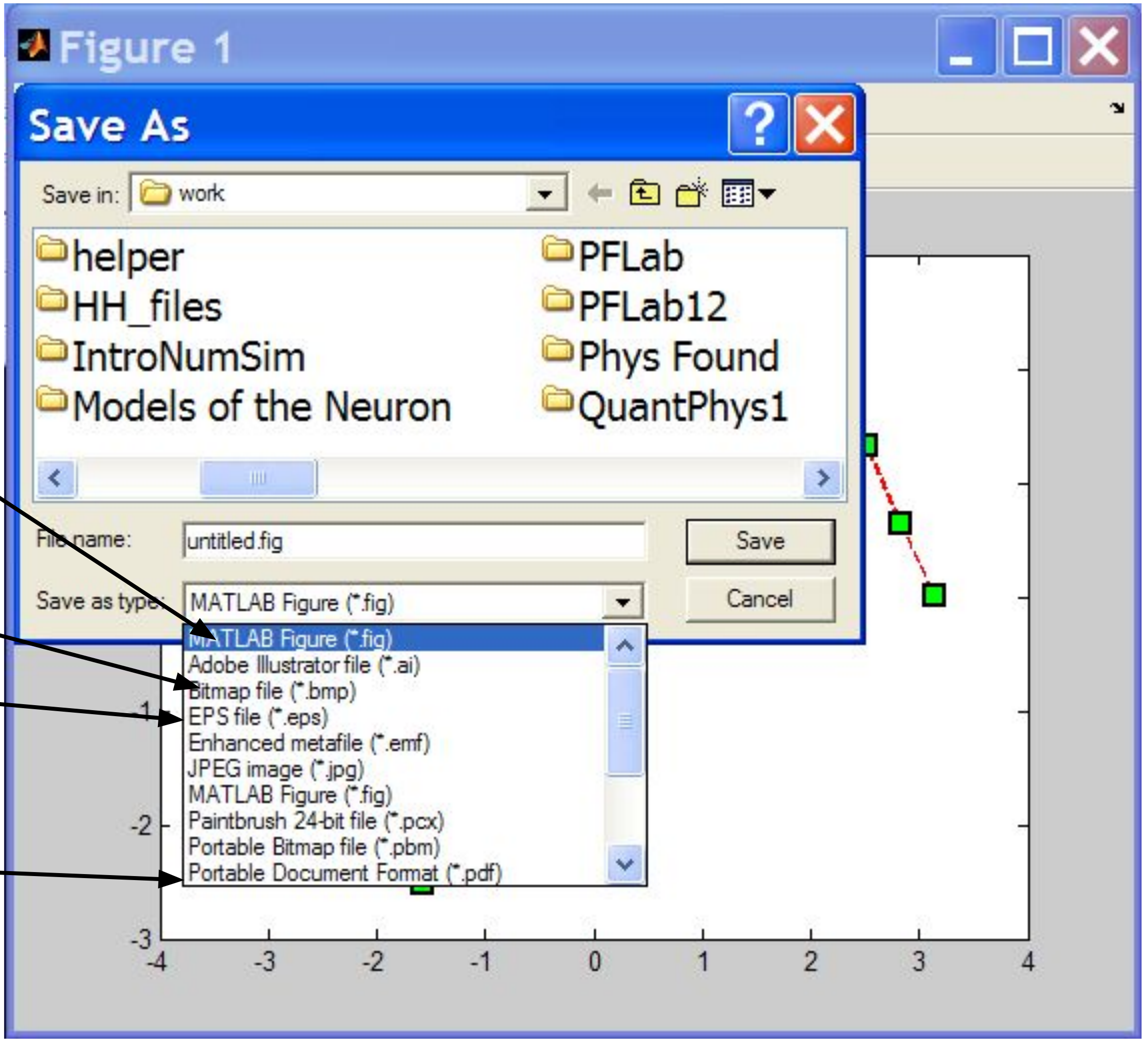
- Figures can be saved in many formats. The common ones are:

.fig preserves all information

.bmp uncompressed image

.eps high-quality scaleable format

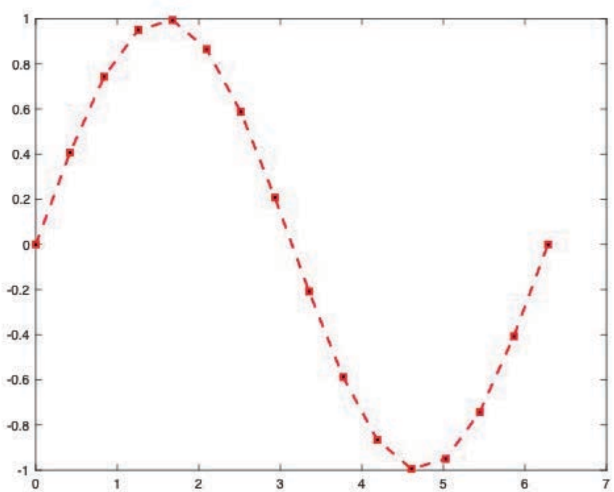
.pdf compressed image



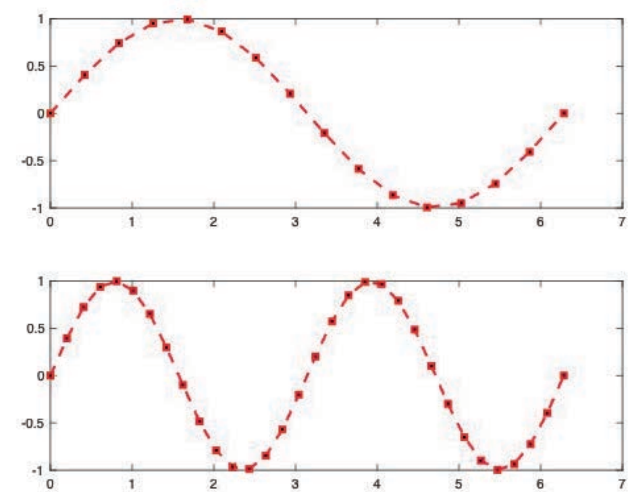
Advanced Plotting: Exercise

- Modify the plot command in your plotSin function to use **squares** as markers and a **dashed red** line of **thickness** 2 as the line. Set the marker face color to be **black** (properties are `LineWidth`, `MarkerFaceColor`)
- If there are 2 inputs, open a new figure with 2 axes, one on top of the other (not side by side), and plot both frequencies (`subplot`)

`plotSin(6)`



`plotSin(1,2)`

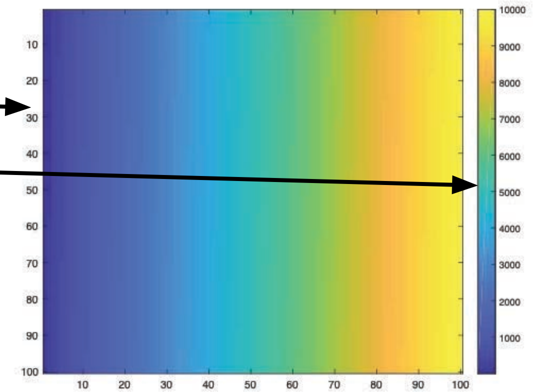


Outline

(1) Functions
(2) Flow Control
(3) Line Plots
(4) Image/Surface Plots
(5) Efficient Codes
(6) Debugging

Visualizing matrices

- Any matrix can be visualized as an image
  - » `mat=reshape(1:10000,100,100);`
  - » `imagesc(mat);`
  - » `colorbar`

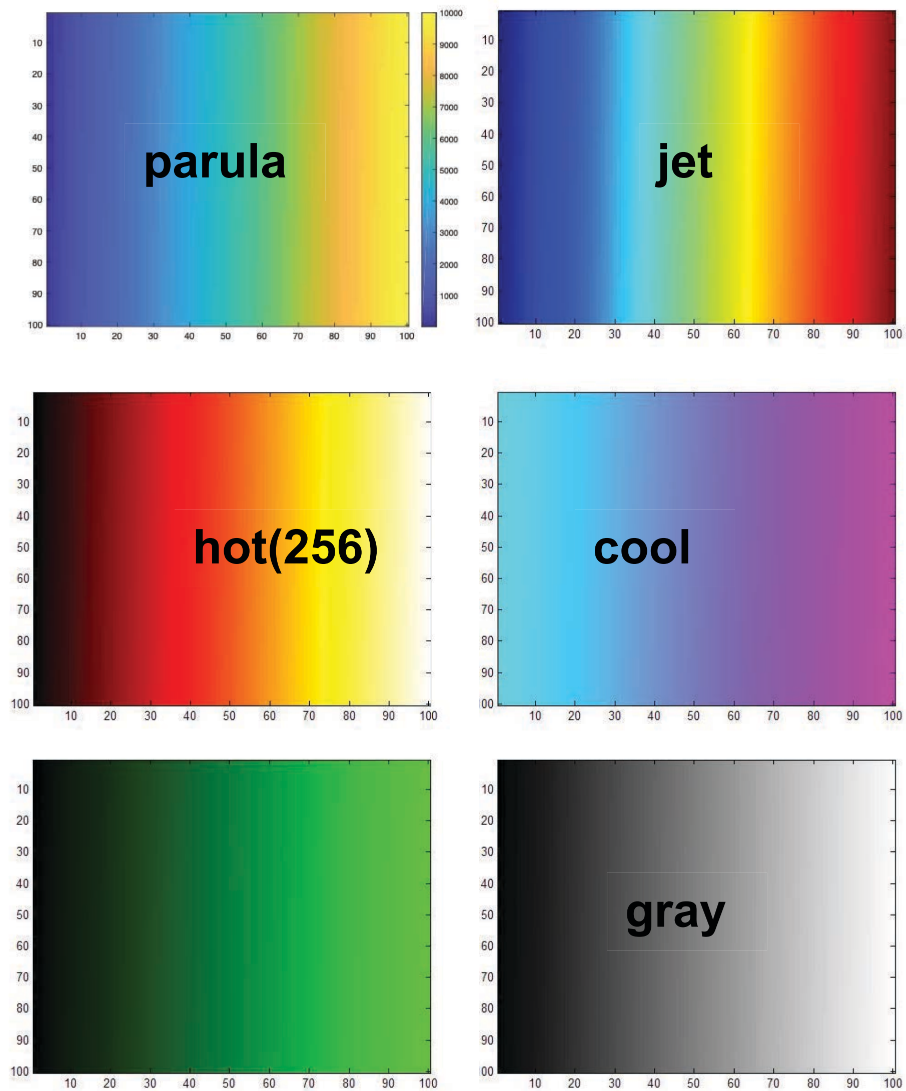


- **imagesc** automatically scales the values to span the entire colormap

- Can set limits for the color axis (analogous to `xlim`, `ylim`)
  - » `caxis([3000 7000])`

Colormaps

- You can change the colormap:
  - » `imagesc(mat)`
    - ➢ default map is parula
  - » `colormap(gray)`
  - » `colormap(cool)`
  - » `colormap(hot(256))`
- See `help hot` for a list
- Can define custom color-map
  - » `map=zeros(256,3);`
  - » `map(:,2)=(0:255)/255;`
  - » `colormap(map);`

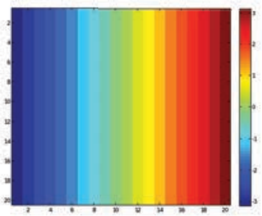


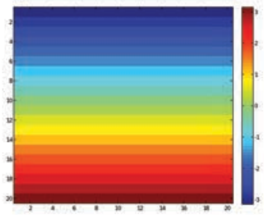
Surface Plots

- It is more common to visualize *surfaces* in 3D
- Example:

$$f(x,y) = sin(x)cos(y)$$
$$x \in [-\pi,\pi]; y \in [-\pi,\pi]$$

- **surf** puts vertices at specified points in space x,y,z, and connects all the vertices to make a surface
- The vertices can be denoted by matrices X,Y,Z
- How can we make these matrices
  - ➢ built-in function: **meshgrid**





surf

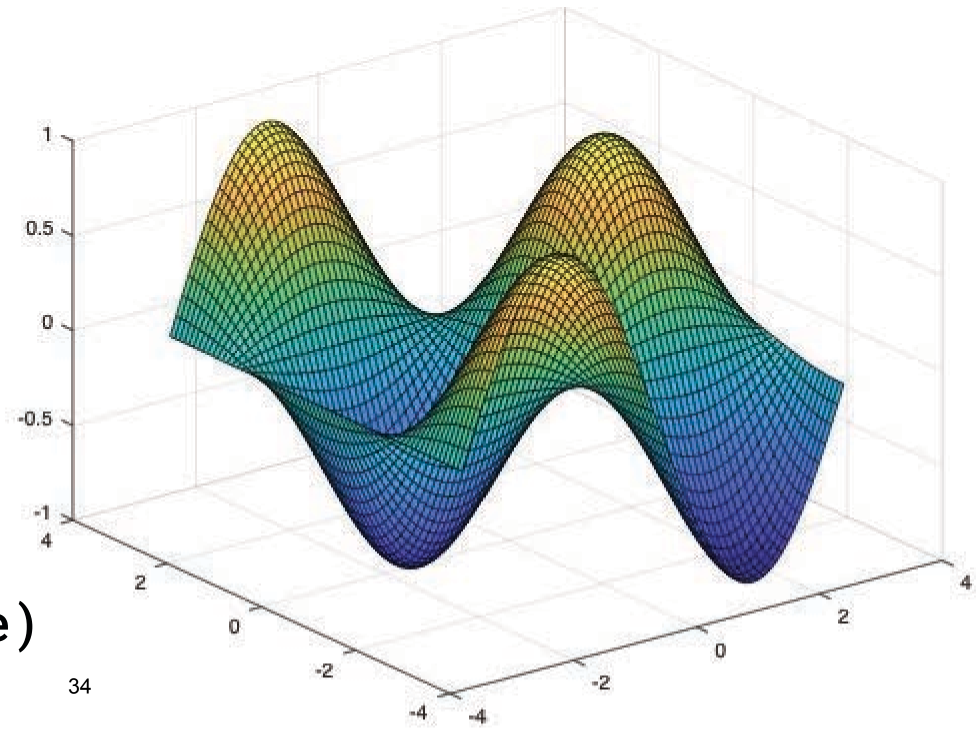
- Make the x and y vectors
  » `x=-pi:0.1:pi;`
  » `y=-pi:0.1:pi;`

- Use meshgrid to make matrices
  » `[X,Y]=meshgrid(x,y);`

- To get function values, evaluate the matrices
  » `Z =sin(X).*cos(Y);`

- Plot the surface
  » `surf(X,Y,Z)`
  » `surf(x,y,Z);`

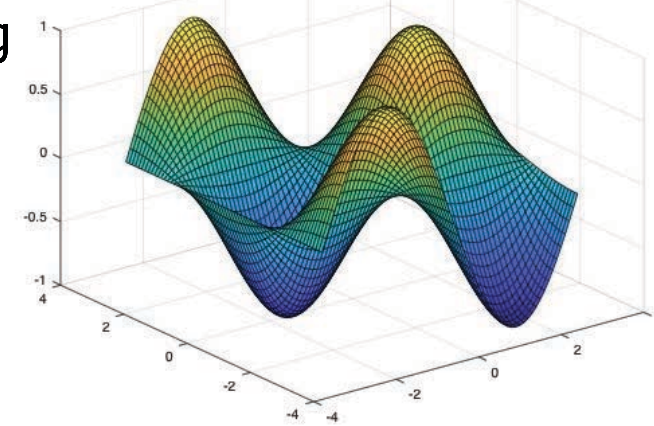
*Try typing **`surf(membrane)`**

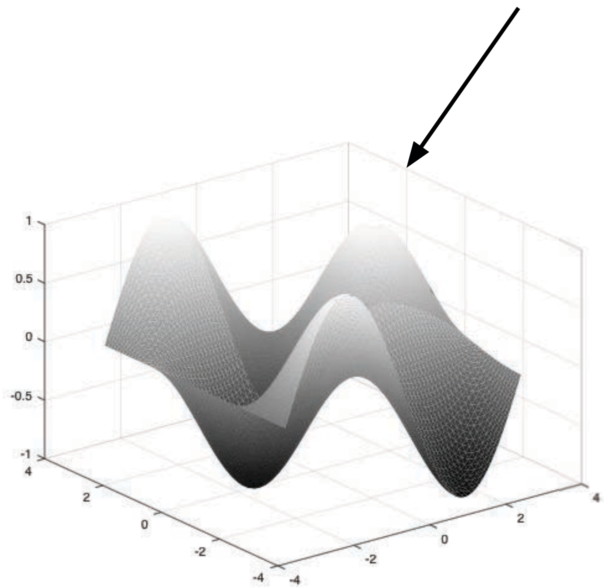


surf Options

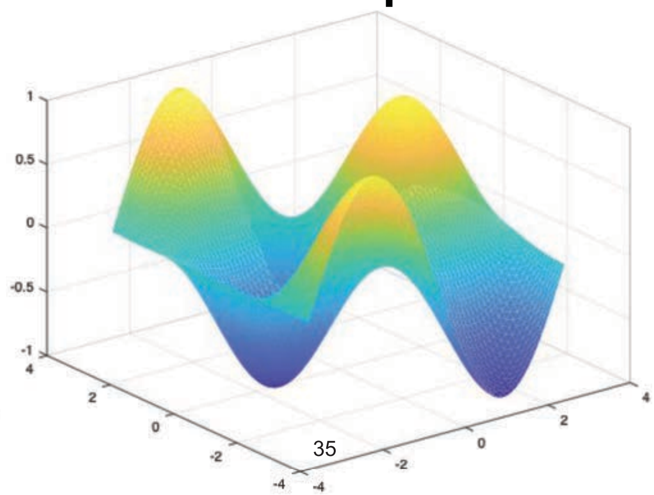
- See **help surf** for more options
- There are three types of surface shading
  - » `shading faceted`
  - » `shading flat`
  - » `shading interp`
- You can also change the colormap
  - » `colormap(gray)`

faceted

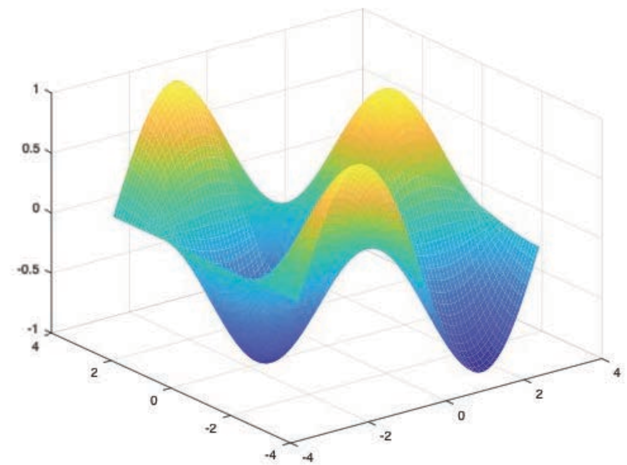




interp

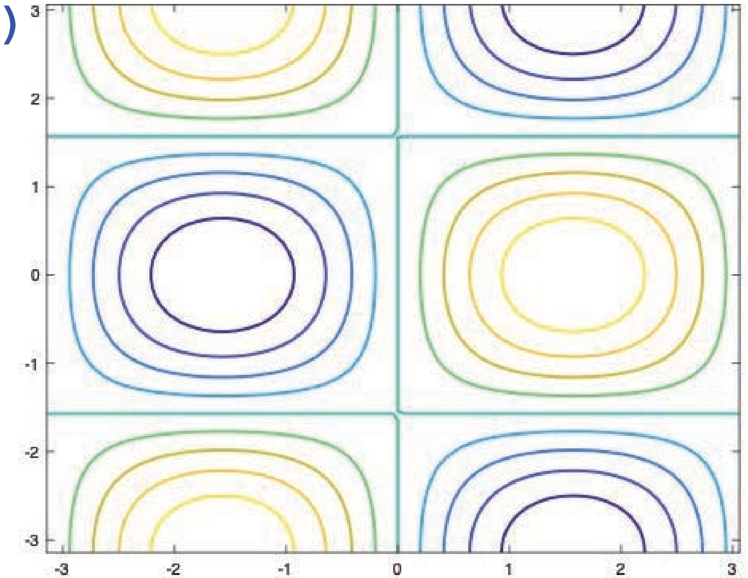


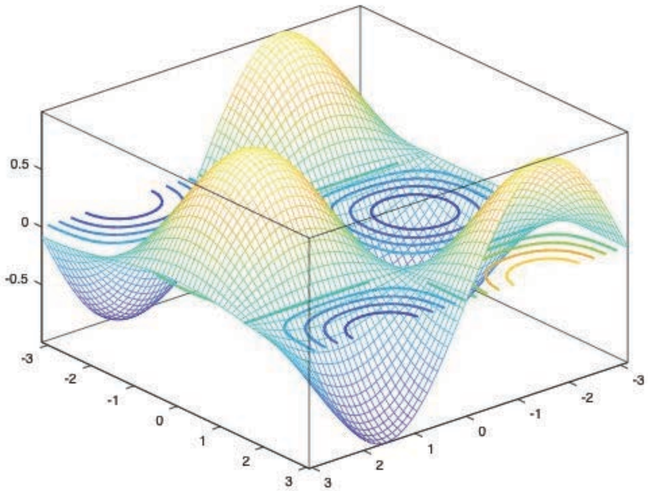
flat



contour

- You can make surfaces two-dimensional by using contour
  - » `contour(X,Y,Z,'LineWidth',2)`
    - ➢ takes same arguments as surf
    - ➢ color indicates height
    - ➢ can modify linestyle properties
    - ➢ can set colormap
  - » `hold on`
  - » `mesh(X,Y,Z)`





Exercise: 3-D Plots

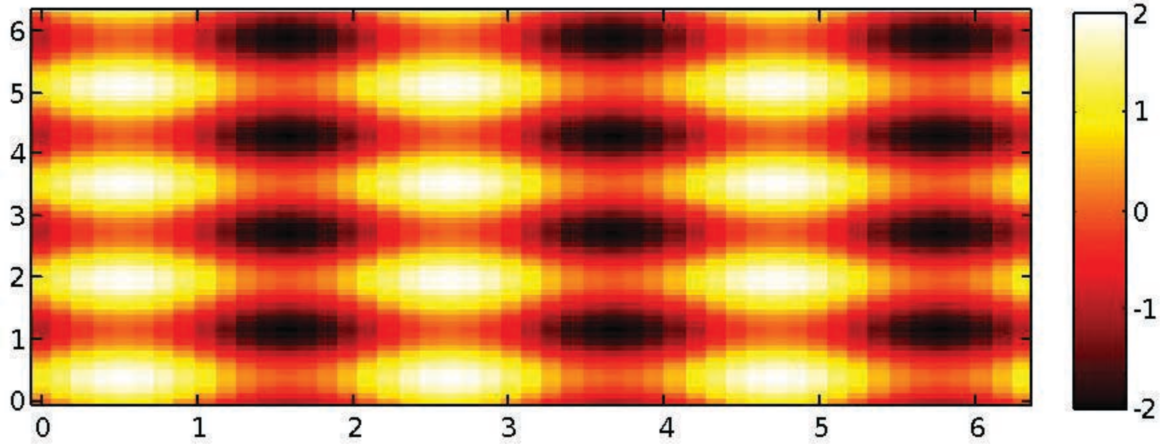
- Modify `plotSin` to do the following:
- If two inputs are given, evaluate the following function:

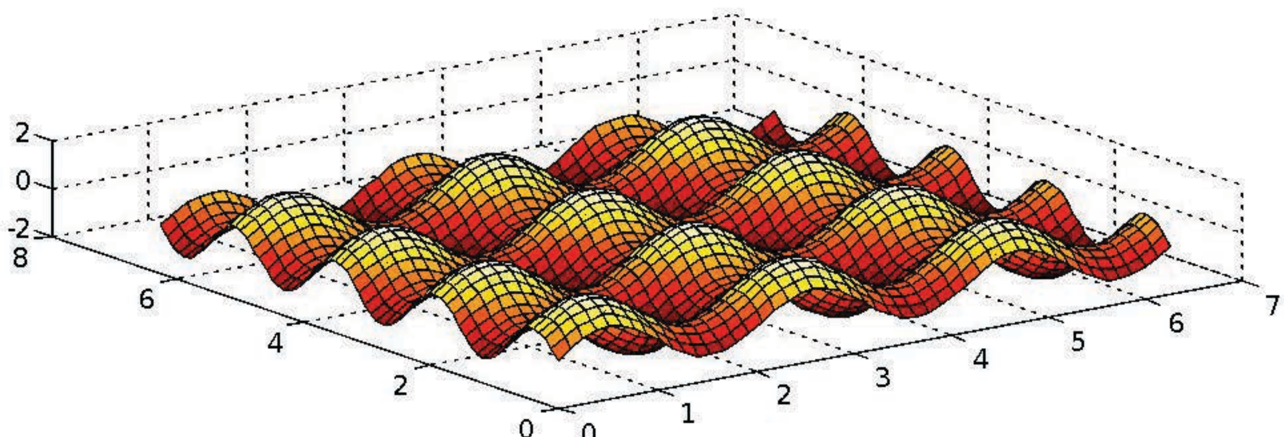
$$Z = \sin(f_1 x) + \sin(f_2 y)$$

- y should be just like x, but using f2. (use `meshgrid` to get the X and Y matrices)
- In the top axis of your subplot, display an image of the Z matrix. Display the colorbar and use a `hot` colormap. Set the axis to xy (`imagesc`, `colormap`, `colorbar`, `axis`)
- In the bottom axis of the subplot, plot the 3-D surface of Z (`surf`)

Exercise: 3-D Plots

`plotSin(3,4)` generates this figure





Specialized Plotting Functions

- MATLAB has a lot of specialized plotting functions
- **polar**-to make polar plots
  - » `polar(0:0.01:2*pi,cos((0:0.01:2*pi)*2))`
- **bar**-to make bar graphs
  - » `bar(1:10,rand(1,10));`
- **quiver**-to add velocity vectors to a plot
  - » `[X,Y]=meshgrid(1:10,1:10);`
  - » `quiver(X,Y,rand(10),rand(10));`
- **stairs**-plot piecewise constant functions
  - » `stairs(1:10,rand(1,10));`
- **fill**-draws and fills a polygon with specified vertices
  - » `fill([0 1 0.5],[0 0 1],'r');`
- see help on these functions for syntax
- **doc specgraph** – for a complete list

Outline

(1) Functions
(2) Flow Control
(3) Line Plots
(4) Image/Surface Plots
(5) **Efficient codes**
(6) Debugging

find

- **find** is a very important function
  - ➢ Returns indices of nonzero values
  - ➢ Can simplify code and help avoid loops

- Basic syntax: index=find(cond)

```
» x=rand(1,100);
» inds = find(x>0.4 & x<0.6);
```

`inds` contains the indices at which x has values between 0.4 and 0.6. This is what happens:

x>0.4 returns a vector with 1 where true and 0 where false

x<0.6 returns a similar vector

& combines the two vectors using logical **and** operator

find returns the indices of the 1's

Example: Avoiding Loops

- Given x= sin(linspace(0,10*pi,100)), how many of the entries are positive?

Using a loop and if/else

```
count=0;
for n=1:length(x)
    if x(n)>0
        count=count+1;
    end
end
```

Being more clever

```
count=length(find(x>0));
```

Is there a better way?!

length(x)	Loop time	Find time
100	0.01	0
10,000	0.1	0
100,000	0.22	0
1,000,000	1.5	0.04

- Avoid loops!
- Built-in functions will make it faster to write and execute

Vectorization

- Avoid loops
  - This is referred to as vectorization
- Vectorized code is more efficient for MATLAB
- Use indexing and matrix operations to avoid loops
- For instance, to add every two consecutive terms:

Vectorization

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  - ➢ This is referred to as vectorization
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- Use indexing and matrix operations to avoid loops
- For instance, to add every two consecutive terms:

```
» a=rand(1,100);
» b=zeros(1,100);
» for n=1:100
»     if n==1
»         b(n)=a(n);
»     else
»         b(n)=a(n-1)+a(n);
»     end
» end
```

  - ➢ Slow and complicated

Vectorization

- Avoid loops
  - ➢ This is referred to as vectorization
- Vectorized code is more efficient for MATLAB
- Use indexing and matrix operations to avoid loops
- For instance, to add every two consecutive terms:

```
» a=rand(1,100);
» b=zeros(1,100);
» for n=1:100
»     if n==1
»         b(n)=a(n);
»     else
»         b(n)=a(n-1)+a(n);
»     end
» end
```

➢ Slow and complicated

```
» a=rand(1,100);
» b=[0 a(1:end-1)]+a;
```

➢ Efficient and clean. Can also do this using `conv`

Preallocation

- Avoid variables growing inside a loop
- Re-allocation of memory is time consuming
- Preallocate the required memory by initializing the array to a default value
- For example:

```
» for n=1:100
»     res = % Very complex calculation %
»     a(n) = res;
» end
```

  ➢ Variable `a` needs to be resized at every loop iteration

Preallocation

- Avoid variables growing inside a loop
- Re-allocation of memory is time consuming
- Preallocate the required memory by initializing the array to a default value
- For example:

```
» a = zeros(1, 100);
» for n=1:100
»     res = % Very complex calculation %
»     a(n) = res;
» end
```

  - ➢ Variable `a` is only assigned new values. No new memory is allocated

Outline

(1) Functions
(2) Flow Control
(3) Line Plots
(4) Image/Surface Plots
(5) Efficient codes
(6) **Debugging**

Display

- When debugging functions, use **disp** to print messages
  - » `disp('starting loop')`
  - » `disp('loop is over')`
    - ➢ `disp` prints the given string to the command window

- It's also helpful to show variable values
  - » `disp(['loop iteration ' num2str(n)]);`
    - ➢ Sometimes it's easier to just remove some semicolons

Debugging

- To use the debugger, set breakpoints
  - ➢ Click on – next to line numbers in m-files
  - ➢ Each red dot that appears is a breakpoint
  - ➢ Run the program
  - ➢ The program pauses when it reaches a breakpoint
  - ➢ Use the command window to probe variables
  - ➢ Use the debugging buttons to control debugger

Performance Measures

- It can be useful to know how long your code takes to run
  - ➢ To predict how long a loop will take
  - ➢ To pinpoint inefficient code

- You can time operations using **tic**/**toc**:

```
» tic
» Mystery1;
» a=toc;
» Mystery2;
» b=toc;
```

  - ➢ tic resets the timer
  - ➢ Each toc returns the current value in seconds
  - ➢ Can have multiple tocs per tic

Performance Measures

- Example: Sparse matrices

```
» A=zeros(10000); A(1,3)=10; A(21,5)=pi;
» B=sparse(A);
» inv(A);   % what happens?
» inv(B);   % what about now?
```

- If system is sparse, can lead to large memory/time savings

```
» A=zeros(1000); A(1,3)=10; A(21,5)=pi;
» B=sparse(A);
» C=rand(1000,1);
» tic; A\C; toc;   % slow!
» tic; B\C; toc;   % much faster!
```

Performance Measures

- For more complicated programs, use the profiler
  - » **`profile on`**
    - ➢ Turns on the profiler. Follow this with function calls
  - » **`profile viewer`**
    - ➢ Displays gui with stats on how long each subfunction took

Profile Summary

Generated 04-Jan-2006 09:53:26

Number of files called: 19

Filename	File Type	Calls	Total Time	Time Plot
newplot	M-function	1	0.802 s	
gcf	M-function	1	0.460 s	
newplot/ObserveAxesNextPlot	M-subfunction	1	0.291 s	
...matlab/graphics/private/clo	M-function	1	0.251 s	
allchild	M-function	1	0.100 s	
setdiff	M-function	1	0.050 s	

End of Lecture 2

(1) Functions
(2) Flow Control
(3) Line Plots
(4) Image/Surface Plots
(5) Efficient codes
(6) Debugging

Vectorization makes coding fun!

6.057
Introduction to MATLAB

Lecture 3 : Solving Equations, Curve Fitting, and Numerical Techniques

Orhan Celiker

IAP 2019

Outline

(1) **Linear Algebra**
(2) Polynomials
(3) Optimization
(4) Differentiation/Integration
(5) Differential Equations

Systems of Linear Equations

MATLAB makes linear algebra fun!

- Given a system of linear equations
  - x+2y-3z=5
  - -3x-y+z=-8
  - x-y+z=0
- Construct matrices so the system is described by Ax=b

```
» A=[1 2 -3;-3 -1 1;1 -1 1];
» b=[5;-8;0];
```

- And solve with a single line of code!

```
» x=A\b;
```

  - x is a 3x1 vector containing the values of x, y, and z

- **The \ will work with square or rectangular systems.**
- Gives least squares solution for rectangular systems. Solution depends on whether the system is over or underdetermined.

Worked Example: Linear Algebra

- Solve the following systems of equations:

  - System 1:

$$x + 4y = 34$$
$$-3x + y = 2$$

```
» A=[1 4;-3 1];
» b=[34;2];
» rank(A)
» x=inv(A)*b;
» x=A\b;
```

  - System 2:

$$2x - 2y = 4$$
$$-x + y = 3$$
$$3x + 4y = 2$$

```
» A=[2 -2;-1 1;3 4];
» b=[4;3;2];
» rank(A)
```

  - rectangular matrix

```
» x=A\b;
```

  - gives least squares solution

```
» error=abs(A*x1-b)
```

More Linear Algebra

- Given a matrix
  - » `mat=[1 2 -3;-3 -1 1;1 -1 1];`
- Calculate the rank of a matrix
  - » `r=rank(mat);`
    - ➢ the number of linearly independent rows or columns
- Calculate the determinant
  - » `d=det(mat);`
    - ➢ mat must be square; matrix invertible if det nonzero
- Get the matrix inverse
  - » `E=inv(mat);`
    - ➢ if an equation is of the form A*x=b with A a square matrix, x=A\b is (mostly) the same as x=inv(A)*b
- Get the condition number
  - » `c=cond(mat);` (or its reciprocal: `c = rcond(mat);`)
    - ➢ if condition number is large, when solving A*x=b, small errors in b can lead to large errors in x (optimal c==1)

Matrix Decompositions

- MATLAB has many built-in matrix decomposition methods

- The most common ones are
  - » `[V,D]=eig(X)`
    - ➢ Eigenvalue decomposition
  - » `[U,S,V]=svd(X)`
    - ➢ Singular value decomposition
  - » `[Q,R]=qr(X)`
    - ➢ QR decomposition
  - » `[L,U]=lu(X)`
    - ➢ LU decomposition
  - » `R=chol(X)`
    - ➢ Cholesky decomposition (R must be positive definite)

Exercise: Fitting Polynomials

- Find the best second-order polynomial that fits the points: (-1,0), (0,-1), (2,3).

$$a(-1)^2 + b(-1) + c = 0$$
$$a(0)^2 + b(0) + c = -1$$
$$a(2)^2 + b(2) + c = 3$$

Outline

(1) Linear Algebra

(2) Polynomials

(3) Optimization

(4) Differentiation/Integration

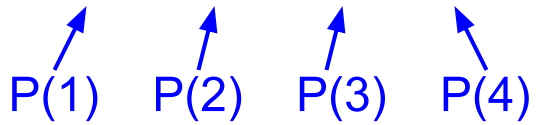
(5) Differential Equations

Polynomials

- Many functions can be well described by a high-order polynomial

- MATLAB represents a polynomials by a vector of coefficients
  - ➢ if vector P describes a polynomial

$$ax^3+bx^2+cx+d$$



- P=[1 0 -2] represents the polynomial x^2-2

- P=[2 0 0 0] represents the polynomial $2x^3$

Polynomial Operations

- P is a vector of length N+1 describing an N-th order polynomial
- To get the roots of a polynomial
  - » **`r=roots(P)`**
    - ➢ r is a vector of length N

- Can also get the polynomial from the roots
  - » **`P=poly(r)`**
    - ➢ r is a vector length N

- To evaluate a polynomial at a point
  - » **`y0=polyval(P,x0)`**
    - ➢ x0 is a single value; y0 is a single value

- To evaluate a polynomial at many points
  - » **`y=polyval(P,x)`**
    - ➢ x is a vector; y is a vector of the same size

Polynomial Fitting

- MATLAB makes it very easy to fit polynomials to data

- Given data vectors X=[-1 0 2] and Y=[0 -1 3]
  - » **p2=polyfit(X,Y,2);**
    - ➢ finds the best (least-squares sense) second-order polynomial that fits the points (-1,0),(0,-1), and (2,3)
    - ➢ see **help polyfit** for more information
  - » **plot(X,Y,'o', 'MarkerSize', 10);**
  - » **hold on;**
  - » **x = -3:.01:3;**
  - » **plot(x,polyval(p2,x), 'r--');**

Exercise: Polynomial Fitting

- Evaluate $y = x^2$ for x=-4:0.1:4.
- Add random noise to these samples. Use **randn**. Plot the noisy signal with **.** markers
- Fit a 2nd degree polynomial to the noisy data
- Plot the fitted polynomial on the same plot, using the same x values and a red line

Outline

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(2) Polynomials
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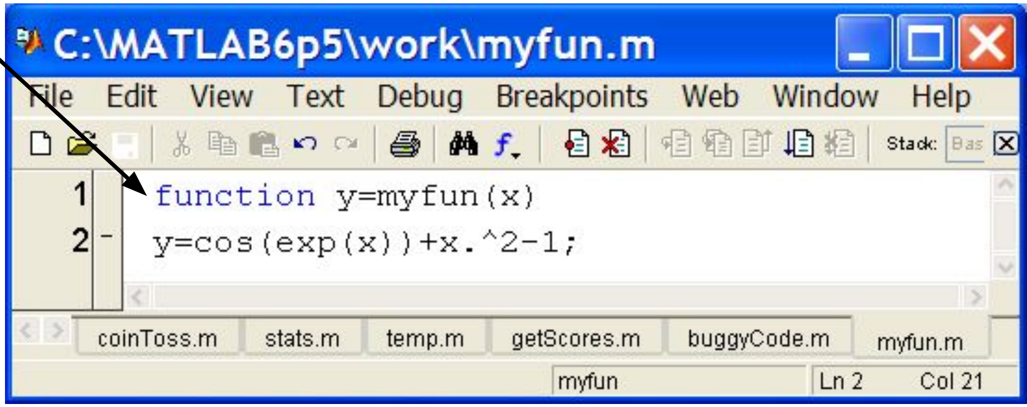
Nonlinear Root Finding

- Many real-world problems require us to solve f(x)=0
- Can use **fzero** to calculate roots for *any* arbitrary function

- **fzero** needs a function passed to it.
- We will see this more and more as we delve into solving equations.

- Make a separate function file
  - » `x=fzero('myfun',1)`
  - » `x=fzero(@myfun,1)`
    - ➢ 1 specifies a point close to where you think the root is



Minimizing a Function

- **fminbnd**: minimizing a function over a bounded interval
  - » `x=fminbnd('myfun',-1,2);`
    - ➢ myfun takes a scalar input and returns a scalar output
    - ➢ myfun(x) will be the minimum of myfun for $-1 \leq x \leq 2$
- **fminsearch**: unconstrained interval
  - » `x=fminsearch('myfun',.5)`
    - ➢ finds the local minimum of myfun starting at x=0.5
- Maximize g(x) by minimizing f(x)=-g(x)
- Solutions may be local!

Anonymous Functions

- You do not have to make a separate function file

```
» x=fzero(@myfun,1)
```

  - What if myfun is really simple?

- Instead, you can make an anonymous function

```
» x=fzero(@(x)(cos(exp(x))+x.^2-1), 1 );
```

input — function to evaluate

```
» x=fminbnd(@(x) (cos(exp(x))+x.^2-1),-1,2);
```

- Can also store the function handle

```
» func=@(x) (cos(exp(x))+x.^2-1);
» func(1:10);
```

Optimization Toolbox

- If you are familiar with optimization methods, use the optimization toolbox
- Useful for larger, more structured optimization problems
- Sample functions (see **`help`** for more info)
  - » **`linprog`**
    - ➢ linear programming using interior point methods
  - » **`quadprog`**
    - ➢ quadratic programming solver
  - » **`fmincon`**
    - ➢ constrained nonlinear optimization

Exercise: Min-Finding

- Find the minimum of the function $f(x) = \cos(4x)\sin(10x)e^{-|x|}$ over the range −π to π. Use **`fminbnd`**.
- Plot the function on this range to check that this is the minimum.

Digression: Numerical Issues

- Many techniques in this lecture use floating point numbers
- **This is an approximation!**

- Examples:
  - » `sin(pi) = ?`
  - » `sin(2 * pi) = ?`
  - » `sin(10e16 * pi) = ?`
    - ➢ Both sin and pi are approximations!

  - » `A = (10e13)*ones(10) + rand(10)`
    - ➢ A is nearly singular, poorly conditioned (see `cond(A)`)
  - » `inv(A)*A = ?`

A Word of Caution

- MATLAB knows no fear!
- Give it a function, it optimizes / differentiates / integrates
  - ➢ That's great! It's so powerful!
- Numerical techniques are powerful **but** not magic
- **Beware of overtrusting the solution!**
  - ➢ You will get an answer, but it may not be what you want
- Analytical forms may give more intuition
  - ➢ Symbolic Math Toolbox

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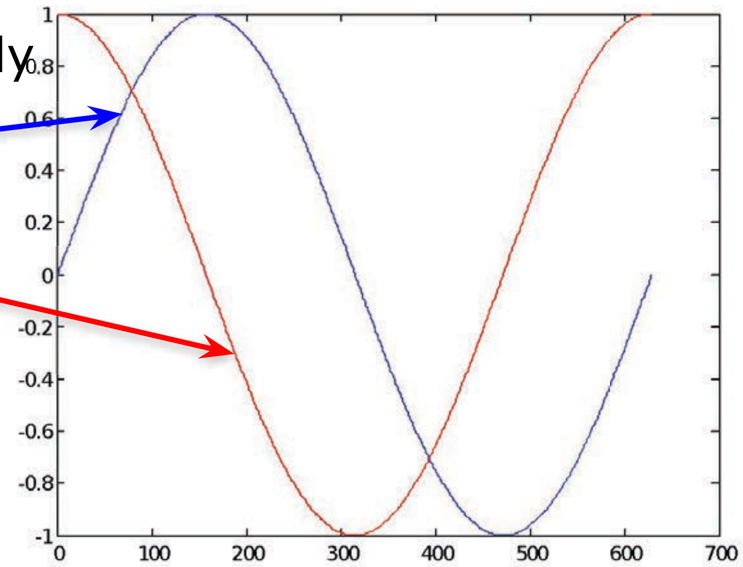
Numerical Differentiation

- MATLAB can 'differentiate' numerically
  ```
  » x=0:0.01:2*pi;
  » y=sin(x);
  » dydx=diff(y)./diff(x);
  ```
  - ➢ diff computes the first difference



- Can also operate on matrices
  ```
  » mat=[1 3 5;4 8 6];
  » dm=diff(mat,1,2)
  ```
  - ➢ first difference of mat along the 2^{nd} dimension, dm=[2 2;4 -2]
  - ➢ The opposite of **`diff`** is the cumulative sum **`cumsum`**

- 2D gradient
  ```
  » [dx,dy]=gradient(mat);
  ```

- Higher derivatives / complicated problems: Fit spline (see **help**)

Numerical Integration

- MATLAB contains common integration methods
- Adaptive Simpson's quadrature (input is a function)
  - » **`q=quad('myFun',0,10)`**
    - ➢ q is the integral of the function **`myFun`** from 0 to 10
  - » **`q2=quad(@(x) sin(x).*x,0,pi)`**
    - ➢ q2 is the integral of **`sin(x).*x`** from 0 to pi
- Trapezoidal rule (input is a vector)
  - » **`x=0:0.01:pi;`**
  - » **`z=trapz(x,sin(x))`**
    - ➢ z is the integral of sin(x) from 0 to pi
  - » **`z2=trapz(x,sqrt(exp(x))./x)`**
    - ➢ z2 is the integral of $\sqrt{e^x}/x$ from 0 to pi

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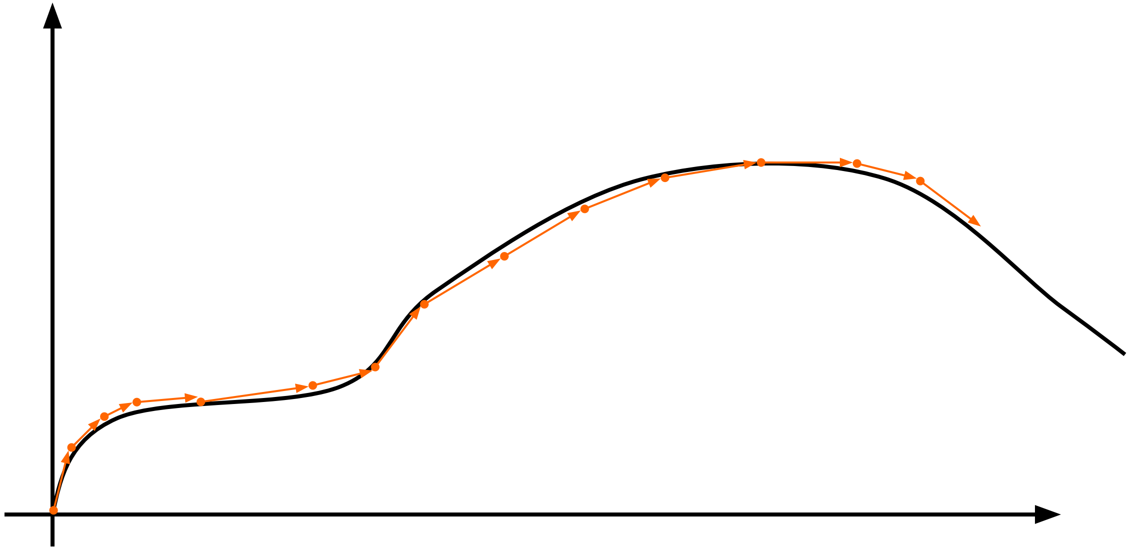
ODE Solvers: Method

- Given a differential equation, the solution can be found by integration:



  - Evaluate the derivative at a point and approximate by straight line
  - Errors accumulate!
  - Variable timestep can decrease the number of iterations

ODE Solvers: MATLAB

- MATLAB contains implementations of common ODE solvers
- Using the correct ODE solver can save you lots of time and give more accurate results
  - » `ode23`
    - ➢ Low-order solver. Use when integrating over small intervals or when accuracy is less important than speed
  - » `ode45`
    - ➢ High order (Runge-Kutta) solver. High accuracy and reasonable speed. Most commonly used.
  - » `ode15s`
    - ➢ Stiff ODE solver (Gear's algorithm), use when the diff eq's have time constants that vary by orders of magnitude

ODE Solvers: Standard Syntax

- To use standard options and variable time step

```
» [t,y]=ode45('myODE',[0,10],[1;0])
```

ODE integrator: 23, 45, 15s

ODE function

Time range

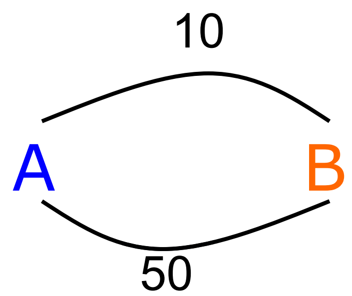
Initial conditions

- Inputs:
  - ➢ ODE function name (or anonymous function). This function should take inputs (t,y), and returns dy/dt
  - ➢ Time interval: 2-element vector with initial and final time
  - ➢ Initial conditions: column vector with an initial condition for each ODE. This is the first input to the ODE function
  - ➢ Make sure all inputs are in the same (variable) order
- Outputs:
  - ➢ t contains the time points
  - ➢ y contains the corresponding values of the variables

ODE Function

- The ODE function must return the value of the derivative at a given time and function value

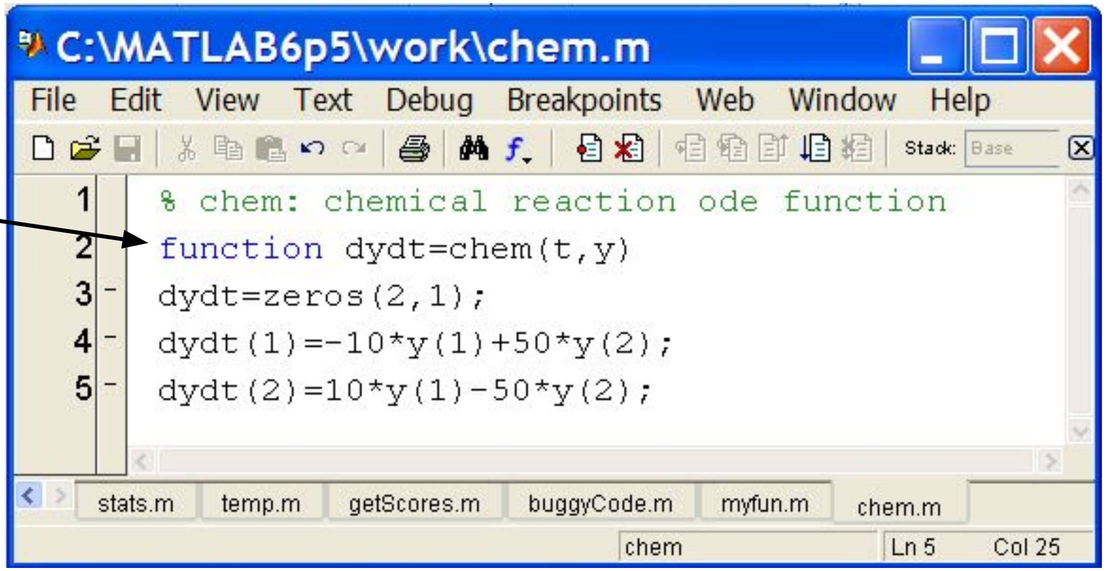
- Example: chemical reaction
  - ➢ Two equations



$$\frac{dA}{dt} = -10A + 50B$$

$$\frac{dB}{dt} = 10A - 50B$$

  - ➢ ODE file:
    - – y has [A;B]
    - – dydt has [dA/dt;dB/dt]

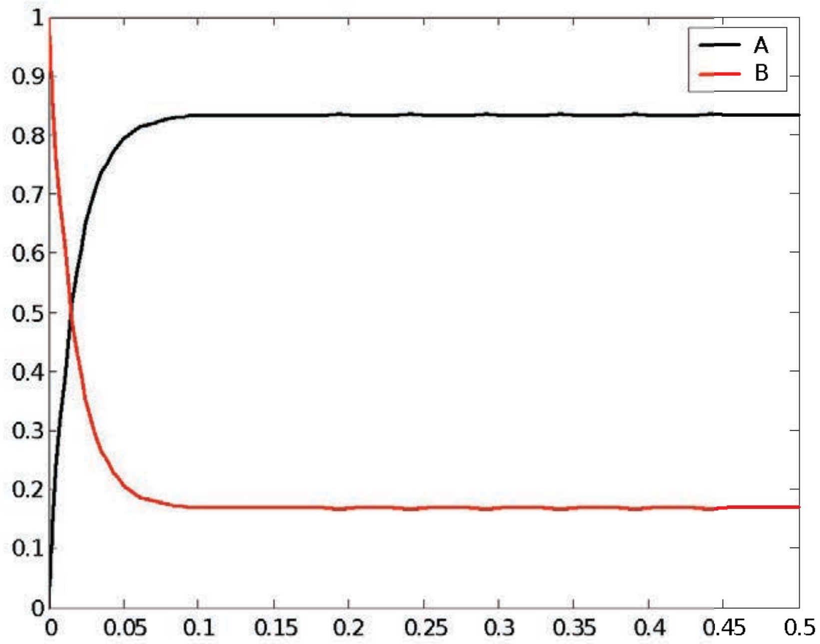


ODE Function: viewing results

- To solve and plot the ODEs on the previous slide:
  - » `[t,y]=ode45('chem',[0 0.5],[0 1]);`
    - ➢ assumes that only chemical B exists initially
  - » `plot(t,y(:,1),'k','LineWidth',1.5);`
  - » `hold on;`
  - » `plot(t,y(:,2),'r','LineWidth',1.5);`
  - » `legend('A','B');`
  - » `xlabel('Time (s)');`
  - » `ylabel('Amount of chemical (g)');`
  - » `title('Chem reaction');`

ODE Function: viewing results

- The code on the previous slide produces this figure



Higher Order Equations

- Must make into a system of first-order equations to use ODE solvers
- Nonlinear is OK!
- Pendulum example:

$$\ddot{\theta} + \frac{g}{L}\sin(\theta) = 0$$

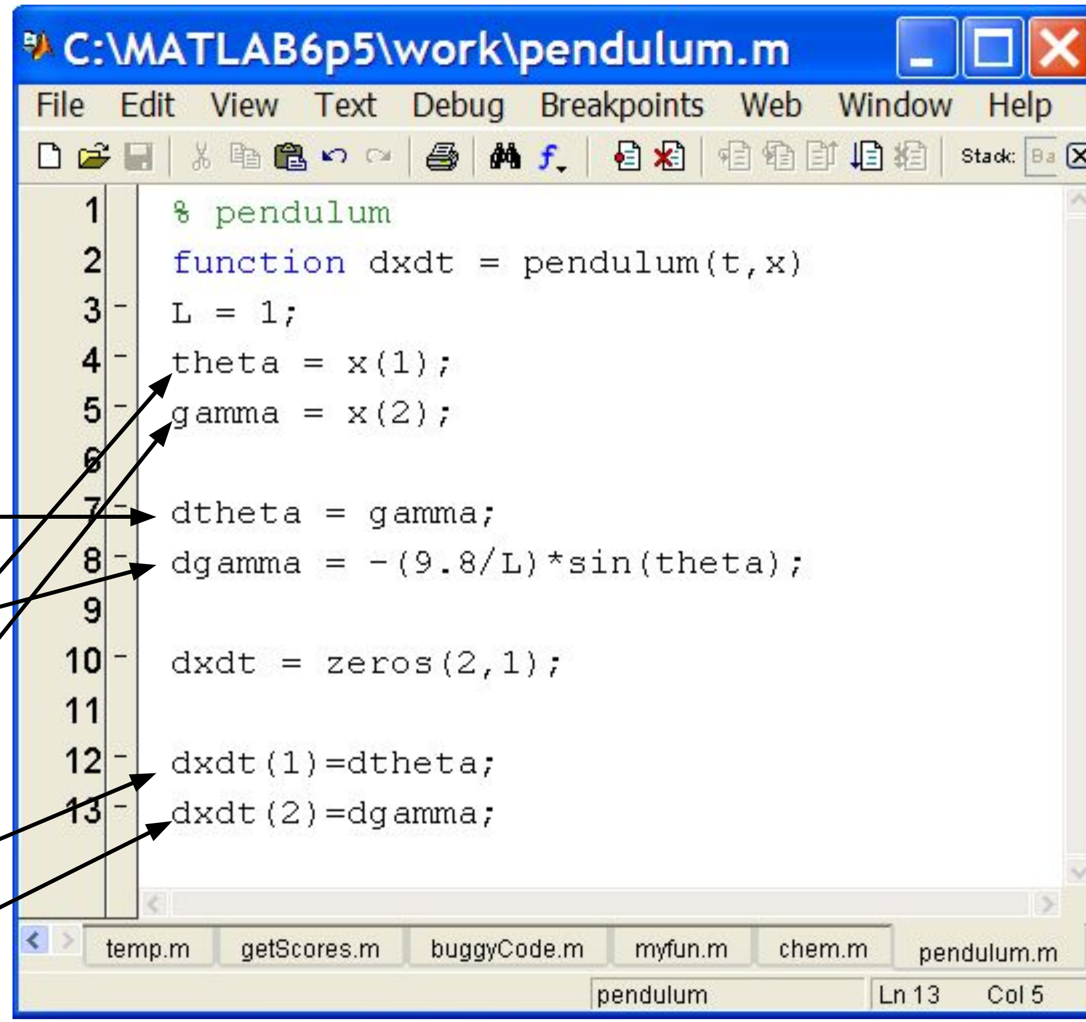
$$\ddot{\theta} = -\frac{g}{L}\sin(\theta)$$

$$let \quad \dot{\theta} = \gamma$$

$$\dot{\gamma} = -\frac{g}{L}\sin(\theta)$$

$$\vec{x} = \begin{bmatrix} \theta \\ \gamma \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \dot{\theta} \\ \dot{\gamma} \end{bmatrix}$$



Plotting the Output

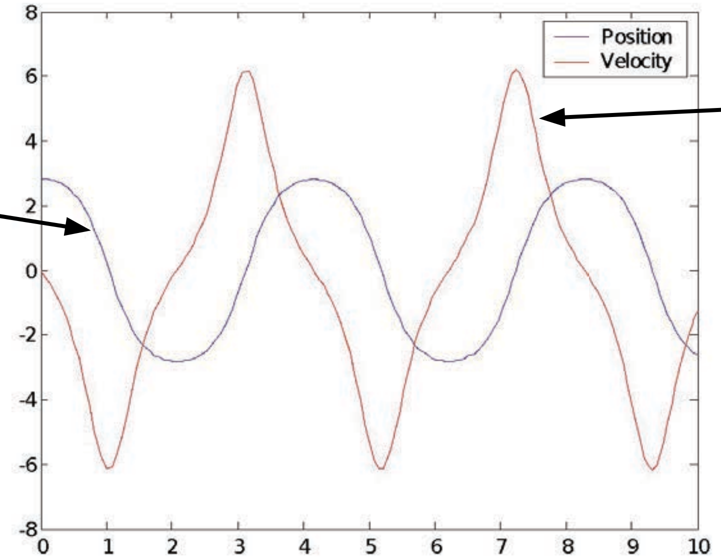
- We can solve for the position and velocity of the pendulum:

```
» [t,x]=ode45('pendulum',[0 10],[0.9*pi 0]);
```

  ➢ assume pendulum is almost horizontal

```
» plot(t,x(:,1));
» hold on;
» plot(t,x(:,2),'r');
» legend('Position','Velocity');
```

Position in terms of angle (rad)



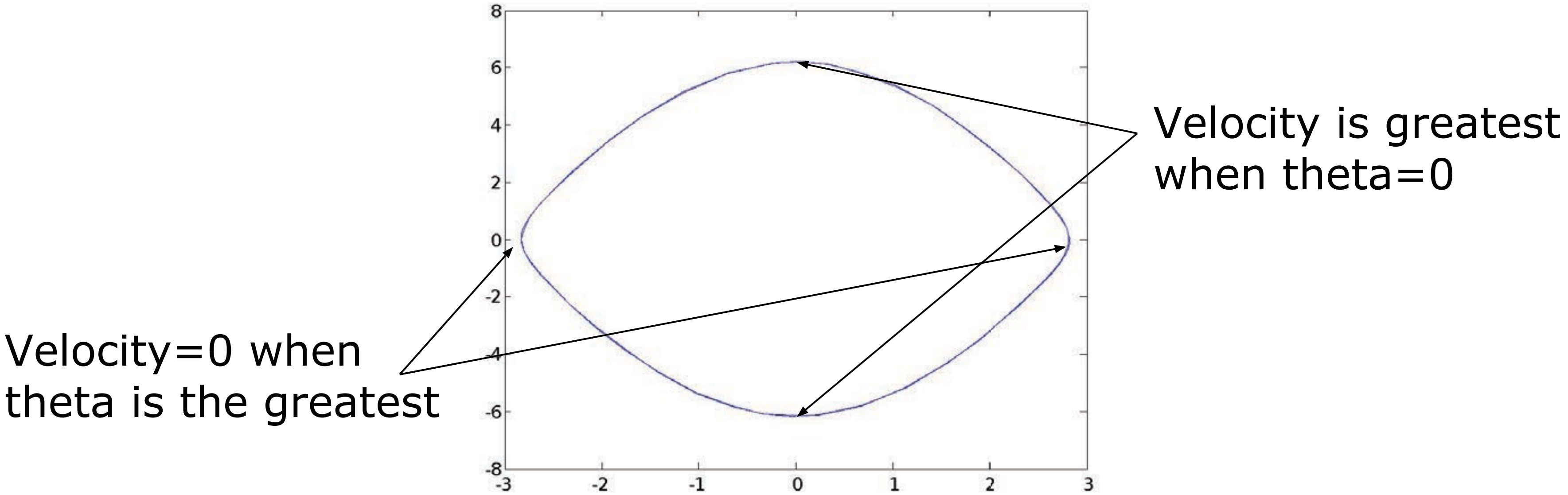
Velocity (m/s)

Plotting the Output

- Or we can plot in the phase plane:

```
» plot(x(:,1),x(:,2));
» xlabel('Position');
» yLabel('Velocity');
```

- The phase plane is just a plot of one variable versus the other:



ODE Solvers: Custom Options

- MATLAB's ODE solvers use a variable timestep
- Sometimes a fixed timestep is desirable
  - » `[t,y]=ode45('chem',[0:0.001:0.5],[0 1]);`
    - ➢ Specify timestep by giving a vector of (increasing) times
    - ➢ The function value will be returned at the specified points

- You can customize the error tolerances using odeset
  - » `options=odeset('RelTol',1e-6,'AbsTol',1e-10);`
  - » `[t,y]=ode45('chem',[0 0.5],[0 1],options);`
    - ➢ This guarantees that the error at each step is less than `RelTol` times the value at that step, and less than `AbsTol`
    - ➢ Decreasing error tolerance can considerably slow the solver
    - ➢ See doc odeset for a list of options you can customize

Exercise: ODE

- Use `ode45` to solve for $y(t)$ on the range t=[0 10], with initial condition $y(0)=10$ and $dy/dt = -t\,y/10$
- Plot the result.

Exercise: ODE

- Use `ode45` to solve for $y(t)$ on the range t=[0 10], with initial condition $y(0)=10$ and $dy/dt=-t\,y/10$
- Plot the result.

- Make the following function

```
» function dydt=odefun(t,y)
» dydt=-t*y/10;
```

- Integrate the ODE function and plot the result

```
» [t,y]=ode45('odefun',[0 10],10);
```

- Alternatively, use an anonymous function

```
» [t,y]=ode45(@(t,y) -t*y/10,[0 10],10);
```

- Plot the result

```
» plot(t,y);xlabel('Time');ylabel('y(t)');
```

Exercise: ODE

- The integrated function looks like this: