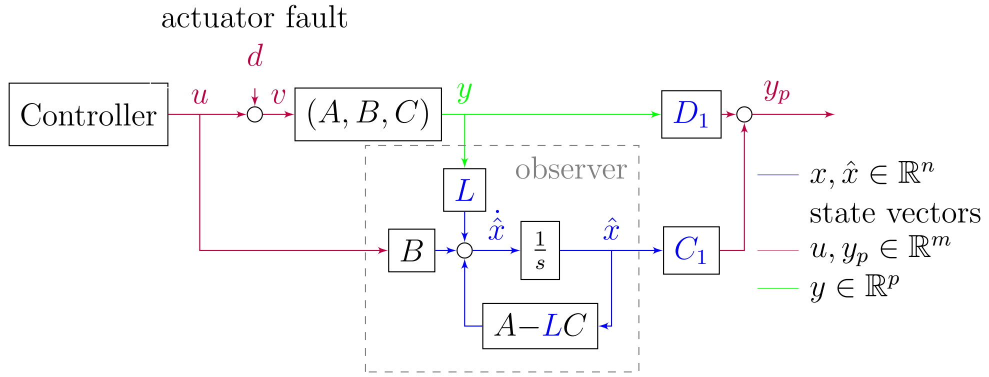
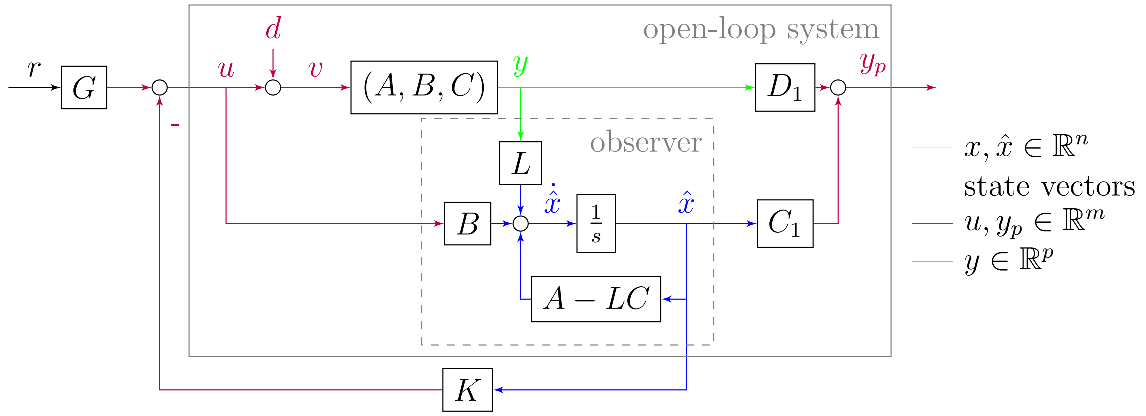
1 MIMO, $u \to y_p$ passivization, feedback equivalence with a passive system

Having an LTI MIMO system, which is somehow fed back through a controller (either tuned by output vector or by the full state vector). The actuator is assumed to be faulty. We intend to detect its fault using system inversion.]. However, the system is not invertible, since its zeros are unstable and/or its vector relative degree (v.r.d.) is more than 1, therefore, we augment the system with an additional (linear) dynamics, which is tuned by the system's output y and the designed control input u, hence its resemblance to an observer.



The goal is to chose matrices L, C_1, D_1 such that the system $u \to y_p$ be feedback equivalent to a passive system and the v.r.d. of $u \to y_p$ be 1.



A K_x rossz ötlet volt!

1.1 Numerical example – MIMO

Having an LTI system with matrices:

$$A = \begin{pmatrix} 6.1 & 1.4 & -0.33 & -1.8 & 0.88 & -0.88 \\ -3.2 & 0.5 & 0.27 & 0.14 & -0.068 & 0.068 \\ 1.4 & -1.5 & 0.58 & 0.14 & -0.07 & 0.07 \\ -0.76 & -1.2 & -0.13 & 0.85 & 1.1 & -1.1 \\ 0.38 & 0.61 & 0.067 & 2.7 & 3.6 & -1.6 \\ -0.38 & -0.61 & -0.067 & 0.59 & 1.7 & 0.3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1.6 & 1 \\ 0.82 & 2 \\ -0.82 & 0 \end{pmatrix} \tag{1}$$

$$C = \begin{pmatrix} -0.15 & -0.24 & -0.026 & -0.2 & 0.6 & -0.6 \\ 0.39 & 0.17 & -1 & 0.58 & 0.21 & -1.2 \end{pmatrix}$$

Its transfer function and its zeros are:

$$H(s) = \begin{pmatrix} \dfrac{s-1}{(s+1)(s-2)} & \dfrac{1}{s-3} \\ \dfrac{(s+2)(s-7)}{(s-5)(s-1)(s-2)} & \dfrac{s-6}{(s-2)(s-3)} \end{pmatrix} \tag{2}$$

$$\texttt{tzero(H)} = \begin{pmatrix} 7.2854 & 0.0214 & 3 & 2 & 1.8361 \end{pmatrix}$$

This system does not have a relative degree $\{1, 1\}$, since

$$\dot{y} = CAx + CBu, \quad CB = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{3}$$

2 Separation principle $K, L = \texttt{place}$, $G = I_m$, $C_1, D_1 = \texttt{sdpvar}$

$$\text{OLS:} \quad \begin{cases} \dot{x} = Ax + Bu \\ \dot{\hat{x}} = LCx + (A - LC)\hat{x} + Bu \\ y_p = D_1 C x + C_1 \hat{x} \end{cases} \Rightarrow \begin{cases} \widehat{A}_o = \begin{pmatrix} A & 0 \\ LC & A - LC \end{pmatrix}, \quad \widehat{B}_o = \begin{pmatrix} B \\ B \end{pmatrix} \\ \widehat{C}_o = \begin{pmatrix} D_1 C & C_1 \end{pmatrix} \end{cases} \tag{4}$$

$$\text{CLS:} \quad u = -KX + Gu \Rightarrow \begin{cases} \widehat{A} = \begin{pmatrix} A & 0 \\ LC & A - LC \end{pmatrix} - BK, \quad \widehat{B} = \widehat{B}_o G \\ \widehat{C} = \begin{pmatrix} D_1 C & C_1 \end{pmatrix} \end{cases} \tag{5}$$

$$\dot{V}(X) \le r^T y_p + y_p^T r - y_p^T W y_p \,, \quad \text{where } W \succ 0,\ V(X) = X^T P X,\ P = P^T \succ 0 \tag{6}$$

$$\begin{pmatrix} X \\ r \end{pmatrix}^T \begin{pmatrix} \widehat{A}^T P + P\widehat{A} & P\widehat{B} \\ \widehat{B}^T P & 0 \end{pmatrix} \begin{pmatrix} X \\ r \end{pmatrix} \le \begin{pmatrix} X \\ r \end{pmatrix}^T \begin{pmatrix} -\widehat{C}^T W \widehat{C} & \widehat{C}^T \\ \widehat{C} & 0 \end{pmatrix} \begin{pmatrix} X \\ r \end{pmatrix} \tag{7}$$

Equivalently: (it could be **only** negative **semi**-definite)

$$M_0 = \begin{pmatrix} \widehat{A}^T P + P\widehat{A} + \widehat{C}^T W \widehat{C} & P\widehat{B} - \widehat{C}^T \\ \widehat{B}^T P - \widehat{C} & 0 \end{pmatrix} \preceq 0, \quad \text{but} \not\prec 0 \tag{8}$$

Using Schur's complement lemma: (it could be **only** negative **semi**-definite)

$$M_1 = \begin{pmatrix} \widehat{A}^T P + P\widehat{A} & P\widehat{B} - \widehat{C}^T & \widehat{C}^T \\ \widehat{B}^T P - \widehat{C} & 0 & 0 \\ \widehat{C} & 0 & -W^{-1} \end{pmatrix} \preceq 0, \quad \text{but} \not\prec 0 \quad \text{(LMI)} \tag{9}$$

2.1 Numerical results

Before optimization:

$$p = \begin{pmatrix} -2 & -1.8 & -1.6 & -1.4 & -1.2 & -1 \end{pmatrix}$$

$$L^T = \texttt{place}(A^T, B^T, p)^T = \begin{pmatrix} -9.45 & -82.2 & 206 & 57.3 & -193 & -199 \\ -85.7 & 58.7 & -50.2 & -0.158 & 16 & 16 \end{pmatrix}$$

$$K = \texttt{place}(A, B, p) = \begin{pmatrix} 5.34 & -0.398 & 0.323 & -0.888 & 0.443 & -0.445 \\ -5.05 & 3.6 & -3.13 & 1.61 & 3.27 & -0.0894 \end{pmatrix} \tag{10}$$

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Ezáltal kaptunk egy stabil observert és az observer által megfigyelt állapotot visszacsatolva egy stabil zárt hurkot.

Ezek után válasszuk meg C_1 és D_1-et úgy, hogy a zárt rendszer passzív is legyen, tehát az open-loop rendszer feedback ekvivalens egy passzív rendszerrel és az $r = \{1, ..., 1\}$ by construction teljesül, ezért az open-loop rendszer minimum fázisú.

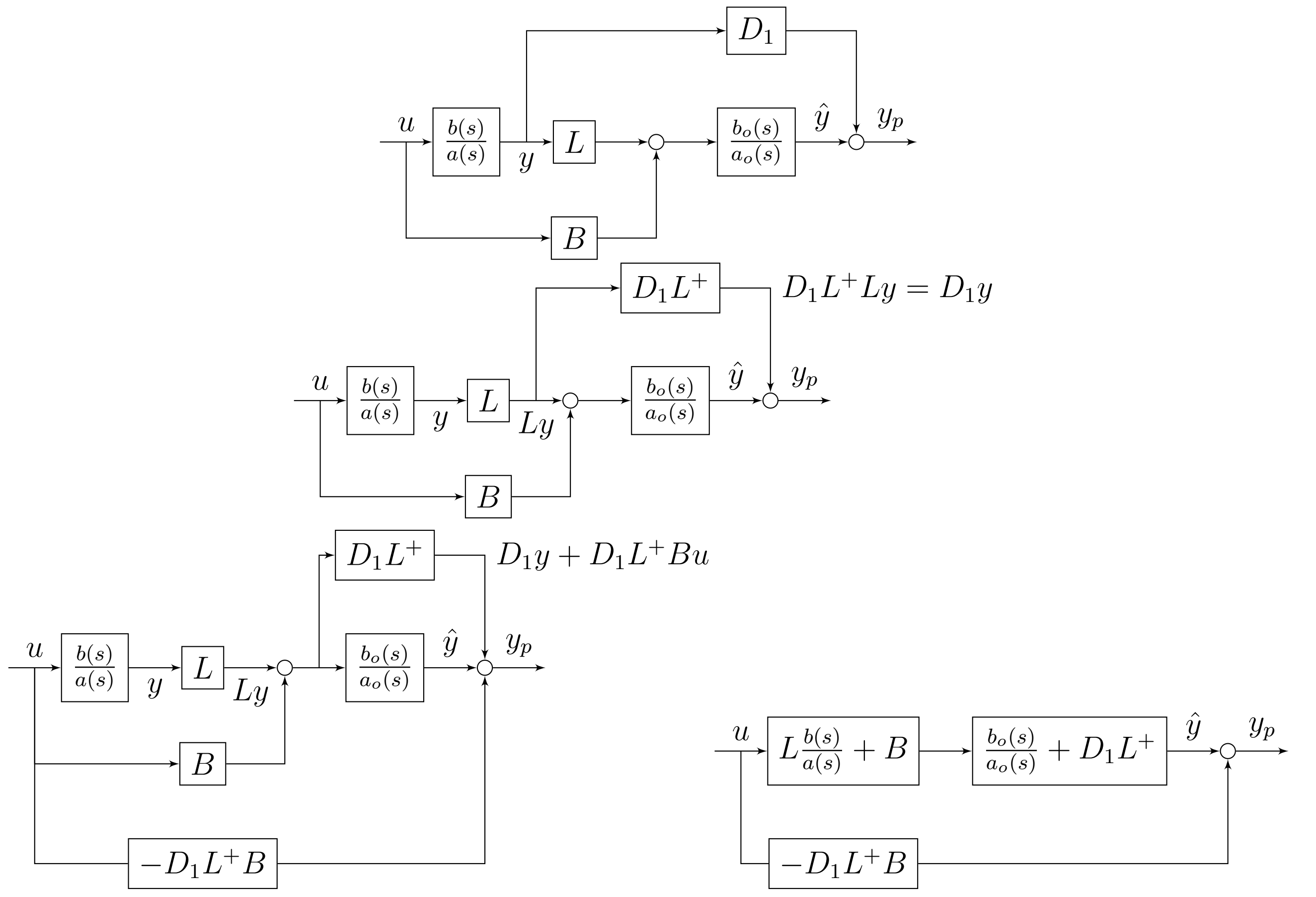
Az optimalizációs eljárás után kapjuk, hogy:

$$C_1 = \begin{pmatrix} 2.81 & -0.758 & -7.06 & 2.13 & 5.98 & -12.3 \\ -1.39 & -2.21 & -0.601 & -1.98 & 6.4 & -6.2 \end{pmatrix}, \quad D_1 = \begin{pmatrix} -7.18 & -6.53 \\ -10.3 & -0.0594 \end{pmatrix} \tag{11}$$

Poles and zeros of the open-loop system:

$$\texttt{POLES} = \begin{pmatrix} 5 & -1 & 2 & 1 & 3 & 2 \end{pmatrix}, \quad \texttt{ZEROS} = \begin{pmatrix} -0.31 + i1.14 & -0.31 - i1.14 & -0.35 & -0.22 \end{pmatrix} \tag{12}$$

3 Megvalósíthatósági tanulmány – operátortartománybeli analízis



Ahol

$$\frac{b_o(s)}{a_o(s)} = C_1 (sI - A + LC)^{-1} \tag{13}$$

Szerintem ez MIMO esetben is jó ($a(s)$ és $a_o(s)$ skalár, minden egyéb mátrix):

$$\begin{aligned} y_p &= D_1 y + \hat{y} = \frac{1}{a(s)} D_1 b(s) u + \frac{1}{a_o(s)} b_o(s) \left(\frac{1}{a(s)} L b(s) + B \right) u \\ G_e(s) &= \frac{1}{a(s)} D_1 b(s) + \frac{1}{a_o(s)} b_o(s) \left(\frac{1}{a(s)} L b(s) + B \right) \\ G_e(s) &= \frac{1}{a(s) a_o(s)} D_1 b(s) a_o(s) + \frac{1}{a(s) a_o(s)} b_o(s) (L b(s) + B a(s)) \\ G_e(s) &= \frac{1}{a(s) a_o(s)} \Big(D_1 b(s) a_o(s) + b_o(s) L b(s) + b_o(s) B a(s) \Big) \end{aligned} \tag{14}$$

Tehát az új számláló:

$$b_e(s) = D_1 b(s) a_o(s) + b_o(s) L b(s) + b_o(s) B a(s) \tag{15}$$

A piros tagok lehetnek instabilak. Kék: megválasztható szabad constans. Mivel $a_o(s)$ skalár:

$$b_e(s) = \Big(a_o(s) D_1 + b_o(s) L \Big) b(s) + b_o(s) B a(s) \leftarrow \text{ legyen stabil} \tag{16}$$

Numerikus példával illusztrálom, hogy ez valóban lehetséges:

$$G(s)=\frac{b(s)}{a(s)}=\frac{s-5}{s^2-3s+2},\quad A=\begin{pmatrix}3 & -2\\ 1 & 0\end{pmatrix},\ B=\begin{pmatrix}2\\ 0\end{pmatrix},\ C=\begin{pmatrix}0.5 & -2.5\end{pmatrix}$$

$$\text{observer: } L\overset{\texttt{place}}{:=}\begin{pmatrix}-13\\ -5\end{pmatrix},\ C_1:=\begin{pmatrix}1 & -1\end{pmatrix}\Rightarrow G_o(s)=\frac{b_o(s)}{a_o(s)}=\frac{1}{s^2+3s+2}\begin{pmatrix}s+9 & -s-25\end{pmatrix}$$

$$b_e(s)=(D_1+2)s^3+(4-2D_1)s^2+(-13D_1-2)s-(10D_1+4)\Big|_{D_1=-1}$$

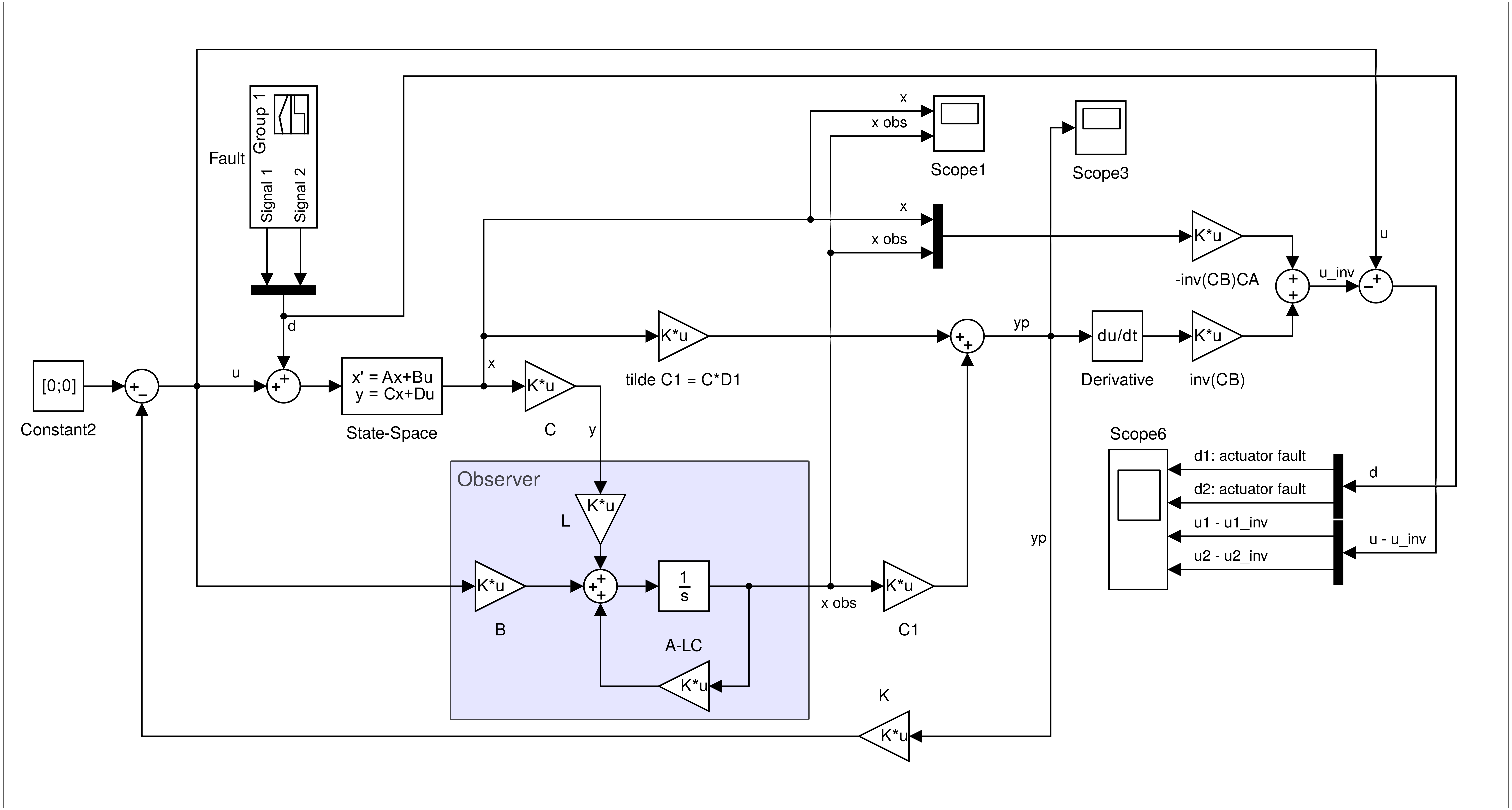
$$=s^3+6s^2+11s+6=(s+3)(s+2)(s+1)$$

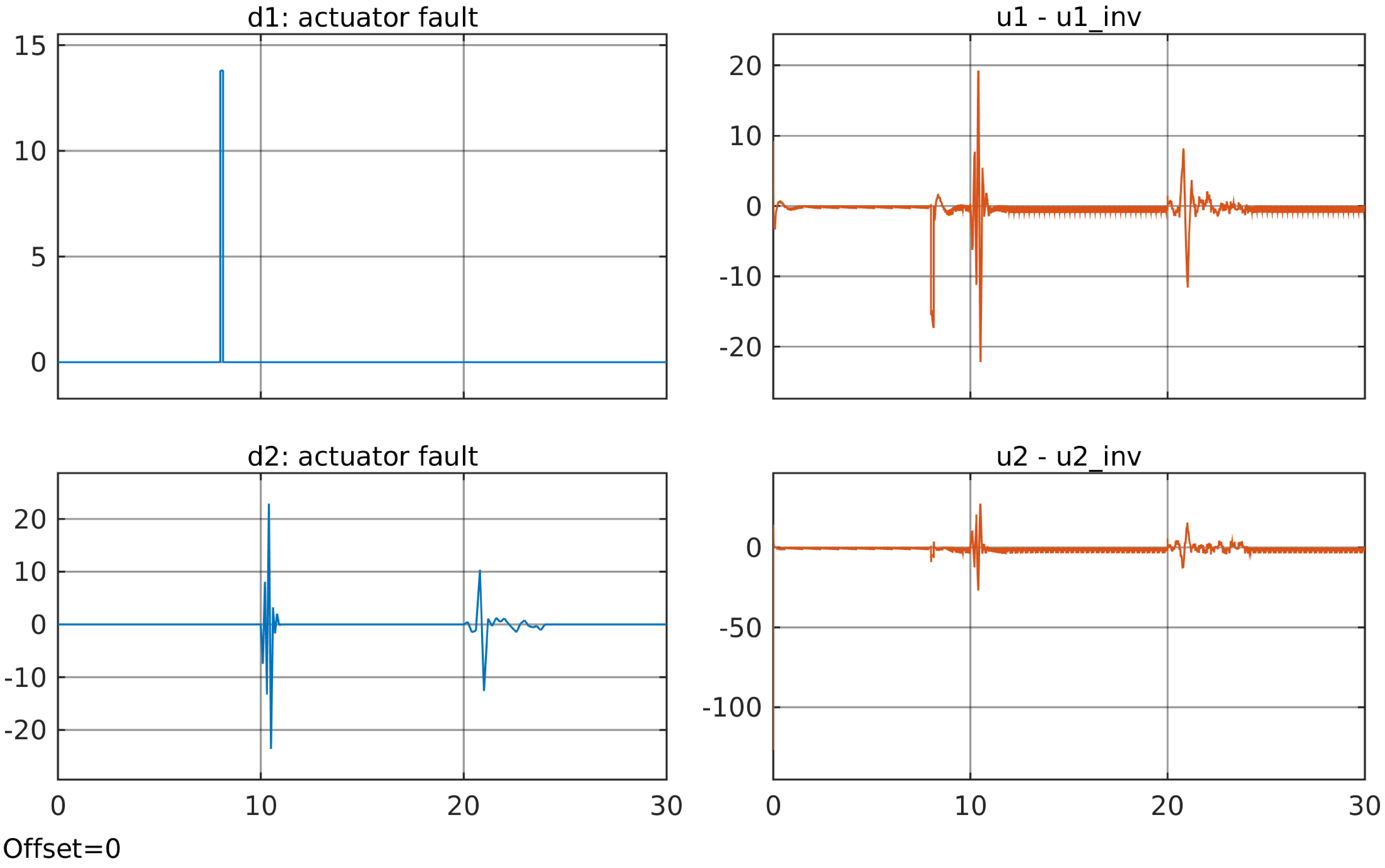
$$\text{eredő átviteli függvény: } G_e(s)=\frac{s+3}{s^2-3s+2} \tag{17}$$

Másik példa (ami a relatív degreet is megjavítja):

$$G(s)=\frac{s-5}{(s-3)(s^2-3s+2)},\quad L^T\overset{\texttt{place}}{:=}\begin{pmatrix}-193 & -212 & -52\end{pmatrix},\ C_1:=\begin{pmatrix}1 & 1 & 1\end{pmatrix},\ D_1:=0.1$$

$$G_e(s)=\frac{s^2+4.1s+3.5}{s^3-6s^2+11s-6} \tag{18}$$


Fault
Group 1
Signal 1
Signal 2
d
Constant2
[0;0]
u
x' = Ax+Bu
y = Cx+Du
State-Space
x
K*u
C
y
tilde C1 = C*D1
Observer
L
B
1
s
A-LC
x obs
C1
K
yp
Scope1
Scope3
-inv(CB)CA
Derivative
du/dt
inv(CB)
u_inv
Scope6
d1: actuator fault
d2: actuator fault
u1 - u1_inv
u2 - u2_inv
u - u_inv



d1: actuator fault
15
10
5
0
u1 - u1_inv
20
10
0
-10
-20
d2: actuator fault
20
10
0
-10
-20
0
10
20
30
u2 - u2_inv
0
-50
-100
0
10
20
30
Offset=0