Computer controlled systems

Lecture 6

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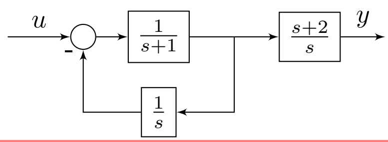
1 Block diagram algebra (Hatásvázlat algebra)

Closed-loop transfer function computation (Eredő átviteli függvény számolása)

Example 1.

1. What is the resulting transfer function $G(s) = ?$ of $\longrightarrow$
Mi az eredő átviteli függvény: $G(s) = ?$



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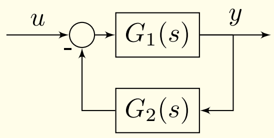
THE RULE:



$$G_e(s) = \frac{G_1(s)}{1 + G_1(s)\,G_2(s)} \tag{1}$$

$$G(s) = \frac{\frac{1}{s+1}\cdot\frac{s+2}{s}}{1+\frac{1}{s}\cdot\frac{1}{s+1}} = \frac{\frac{1}{s+1}}{\frac{s^2+s+1}{s(s+1)}}\cdot\frac{s+2}{s} = \frac{1}{s+1}\cdot\frac{s(s+1)}{s^2+s+1}\cdot\frac{s+2}{s} = \frac{s+2}{s^2+s+1}$$

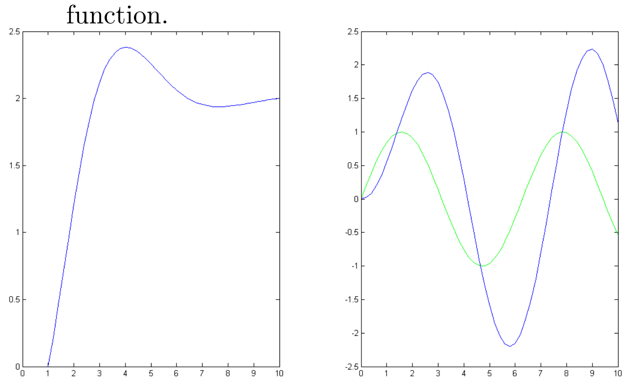
2. Give a possible state space realization for this transfer function!
Controller form:

$$G(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} \quad \overset{\text{Ctrb N.F.}}{\Rightarrow} \quad \begin{aligned} A_c &= \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ C_c &= \begin{bmatrix} b_1 & b_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix} \end{aligned} \tag{2}$$

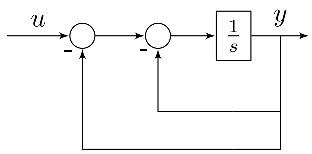
Observer form:

$$G(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} \quad \overset{\text{Obsv N.F.}}{\Rightarrow} \quad \begin{aligned} A_o &= \begin{bmatrix} -a_1 & 1 \\ -a_2 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}, \quad B_o = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ C_o &= \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned} \tag{3}$$

The next figure illustrates the behaviour of the system in case of the unit step function and a sinusoid input function.



Example 2.



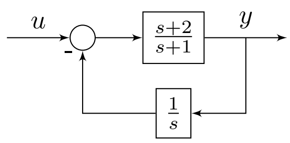
$$H(s) = \frac{1}{s}$$

What is the resulting transfer function $G(s) =$?

$$G_0(s) = \frac{\frac{1}{s}}{1 + \frac{1}{s}} = \frac{1}{s+1}$$

$$G_1(s) = \frac{\frac{1}{s+1}}{1 + \frac{1}{s+1}} = \frac{1}{s+2}$$

Example 3.



What is the resulting transfer function $G(s) =$?

$$G(s) = \frac{\frac{s+2}{s+1}}{1 + \frac{1}{s} \cdot \frac{s+2}{s+1}} = \frac{s(s+2)}{s(s+1) + s + 2} = \frac{s^2 + 2s}{s^2 + 2s + 2}$$

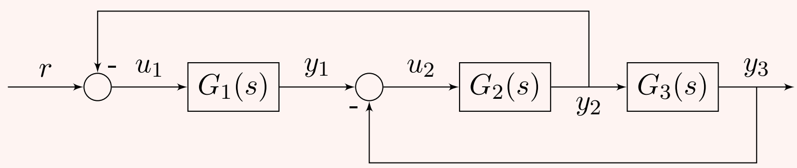
Theoretical questions (minimal computational effort is needed here)

Example 4. Given the following transfer function: $H(s) = \frac{5s^3+2s^2-s+1}{s^4+4s^2-s^2+2s+1}$. Determine whether $H(s)$ is stable or not!

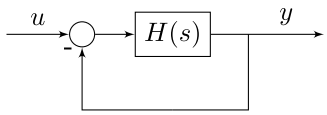
Example 5. Compute the DC-Gain of $H(s) = \frac{s+2}{s^4+3s^2+10s+5}$ in **dB**.

Example 6. (**Computational problem**)

Determine the transfer function $H_{y \to y_3}(s)$ of the following feedback system:



Example 7. Simple negative feedback



Transfer function:

$$\begin{aligned} Y(s) &= H(s)(U(s) - Y(s)) \\ Y(s) + H(s)Y(s) &= H(s)U(s) \\ (1 + H(S))Y(s) &= H(s)U(s) \\ Y(s) &= \frac{H(s)}{1 + H(s)}U(s) \quad \Rightarrow \quad G(s) = \frac{H(s)}{1 + H(s)} \end{aligned}$$

Using this simple negative feedback, determine whether the system is stabilizable or not, if

1. $H(s) = \frac{1}{s}$

$$G(s) = \frac{\frac{1}{s}}{1 + \frac{1}{s}} = \frac{1}{s + 1}$$

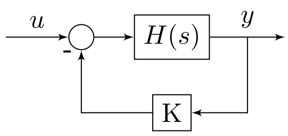
Yes, the system is stabilizable, since the resultant transfer function is stable.

2. $H(s) = \frac{1}{s-2}$

$$G(s) = \frac{\frac{1}{s-2}}{1 + \frac{1}{s-2}} = \frac{1}{s - 1}$$

No, the system is not stabilizable.

Example 8.



Resulting transfer function:

$$G(s) = \frac{H(s)}{1 + KH(s)}$$

$$H(s) = \frac{b(s)}{a(s)} \rightarrow G(s) = \frac{\frac{b(s)}{a(s)}}{1 + K\frac{b(s)}{a(s)}} = \frac{b(s)}{a(s) + Kb(s)}$$

Using this negative feedback with gain K, determine whether the system is stabilizable or not.

1. $H(s) = \frac{1}{s-3}$

$$G(s) = \frac{\frac{1}{s-3}}{1 + K\frac{1}{s-3}} = \frac{1}{s - 3 + K}$$

Therefore, if $K > 3$ the closed loop system is stable.

2. $H(s) = \frac{1}{s-10}$

$$G(s) = \frac{\frac{1}{s-10}}{1 + K\frac{1}{s-10}} = \frac{1}{s - 10 + K}$$

Therefore, if $K > 10$, the closed loop system is again stable.

3. $H(s) = \frac{1}{(s-3)(s-2)}$

$$G(s) = \frac{\frac{1}{(s-3)(s-2)}}{1 + K\frac{1}{(s-3)(s-2)}} = \frac{1}{s^2 - 5s + 6 + K}$$

This system is not stabilizable.

2 Control loop

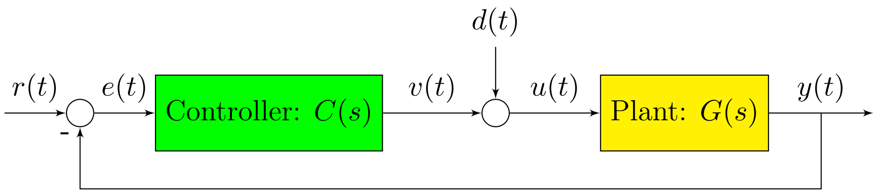


Figure 1

- **Control goal 1. (reference tracking):** to eliminate the error signal $e(t) = r(t) - y(t)$, namely the output signal $y(t)$ converges exponentially to the reference signal $r(t)$. In other words, after a while the output and the reference signal be the same.
- **Control goal 2. (input disturbance reduction):** To lower the transfer between the input disturbance (or actuator fault) $d(t)$ and the output of the error signal $e(t)$, namely: $\left|\frac{E(j\omega)}{D(j\omega)}\right|$ be as smaller as possible.
- Control (or manipulate) signal $v(t)$: the necessary input signal computed by the controller for reference tracking.
- Actuator fault
- The controlled system (Plant) receives the manipulate input $u(t)$ and generates the output signal $y(t)$
- Physical example. Consider a DC motor. Let the input be the current intensity (áramerősség) given to the DC motor, and let the revolution of the motor (fordulatszám) be the output of the DC motor. Then, the error signal will be the difference between the reference revolution and the actual revolution of the DC motor.

Example 9. The control loop presented in Figure 1 can be consider as system with two inputs (reference signal $r(t)$ and input disturbance $d(t)$) and with a single output $y(t)$.

(a) Determine the transfer function $H_{d\to y}(s)$, which is the transfer of $d(t)$ to $y(t)$.

(b) Determine the transfer function $H_{d\to e}(s)$, which is the transfer of $d(t)$ to $e(t)$.

2.1 PID controller

The objective of the PID controller is to eliminate the error signal $e(t) := r(t) - y(t)$, where $r(t)$ is the reference signal, $y(t)$ is the output of the system. In order to do this, the PID controller uses the following signals:

- actual error signal $e(t)$.
- integral of the error signal: $\int_0^t e(\tau)\mathrm{d}\tau$. This constitutes the historical informations of the error signal.
- derivative of the error signal: $\dot{e}(t)$. This gives the actual trend of the error signal.

Therefore, the PID controller *may* contain the following three dynamical components:

- proportional component (P - proportional): $u(t) = K_P \cdot e(t) \qquad H_p(s) = K_P$

- integral component (I - integral): $u(t) = K_I \cdot \int_0^t e(\tau)d\tau \qquad H_I(s) = \frac{K_I}{s}$
- derivative component (D - derivative): $u(t) = K_D \cdot \dot{e}(t) \qquad H_D(s) = s \cdot K_D$

Fontos megjegyezni, hogy a deriváló tag kauzális volta miatt valós rendszerekben a deriváló tagot egy közelítő taggal helyettesítjük.

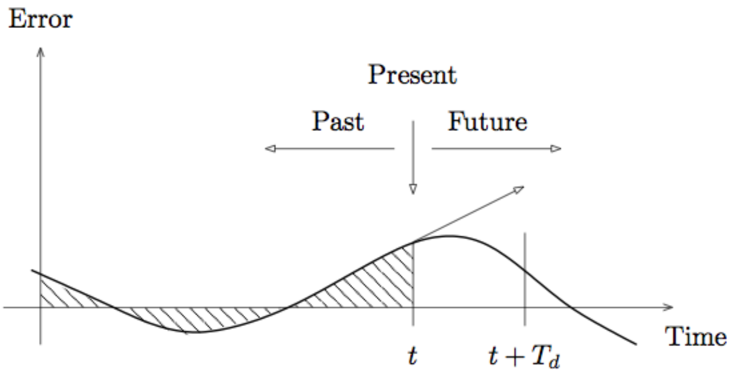


Figure 2

The transfer function of the subsystem (highlighted by the gray dashed box in Figure 3) is the following:

$$H_{PID}(s) = K_p + \frac{K_I}{s} + K_D s = \frac{sK_p + K_I + s^2 K_D}{s}$$

If we use only the P and I components of the PID controller:

$$H_{PI}(s) = K_p + \frac{K_I}{s} = \frac{sK_p + K_I}{s}$$

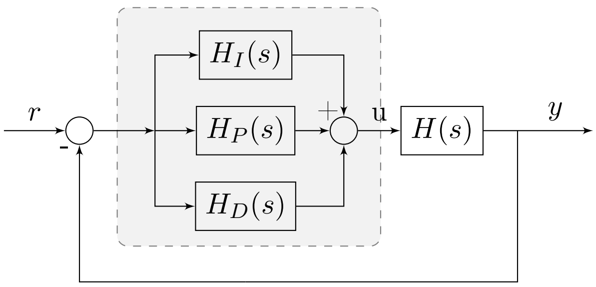


Figure 3

Example 10.

Let us consider the DC motor model, which we mentioned previously:

$$H(s) = \frac{1}{Ms^2 + bs + k} \tag{4}$$

Let $M = 1$, $b = 10$ és $k = 20$
Analyse the response of the system for the unit step function (sse Figure 5). We can see, that the limit at $t \to \infty$ of the output $y(t)$ is much less than the reference signal. This error is called *static error*.
We put into the control loop a proportional term in order to reduce the static error and to obtain a shorter transient (faster rise-time and settling-time).
Helyezzünk a szabályozási körbe egy arányos tagot, ezzel csökkentve a statikus hibát és csökkentve a felfutási időt.

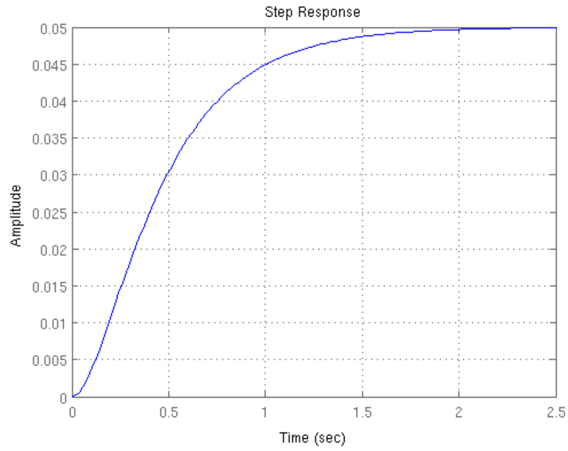


Figure 5. Step response of the uncontrolled system.

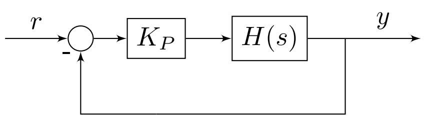


Figure 4. Block diagram of the P controller.

Transfer function of the resulting system:

$$G(s) = \frac{K_p H(S)}{1 + K_P H(s)} = \frac{K_p}{Ms^2 + bs + (k + K_p)} \tag{5}$$

The step response of the system is illustrated in Figure 6.

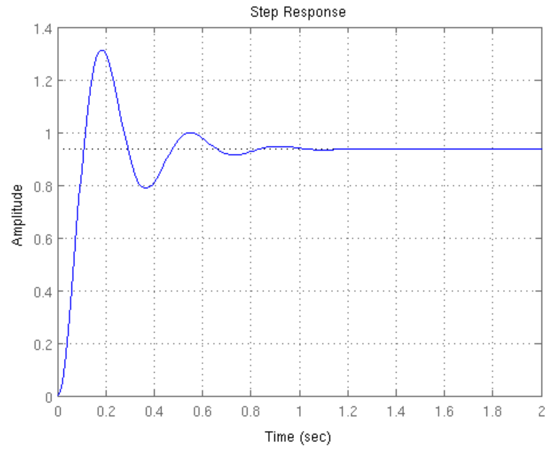


Figure 6. Step response with P controller: $K_p = 300$.

We can see, that the transient time and the static error decreased significantly, however there appears a large overshoot in the step response (the output of the system rises up to 1.3).
Látható, hogy a statikus hiba és a felfutási idő jelentősen csökkent, ugyanakkor jelentős túllövés lett a rendszerválaszban.

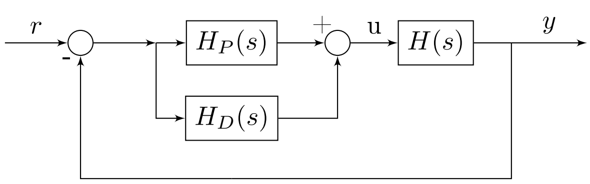


Figure 7. Block diagram of a PD controller.

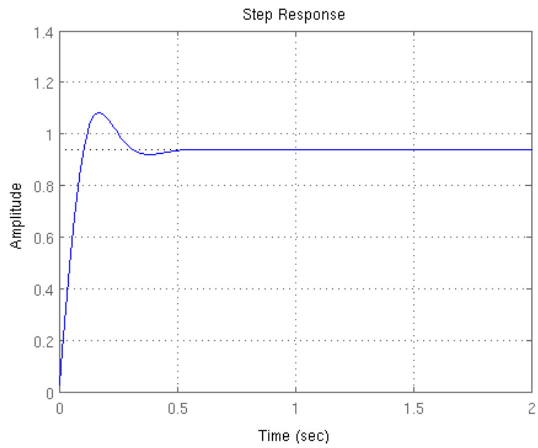


Figure 8. PD controller with $K_p = 30$, $K_i = 70$

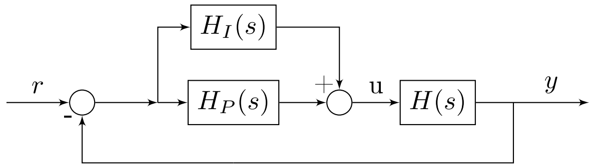


Figure 9. Block diagram of a PI controller.

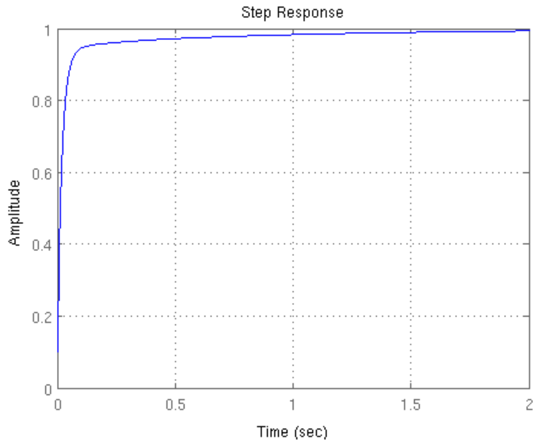
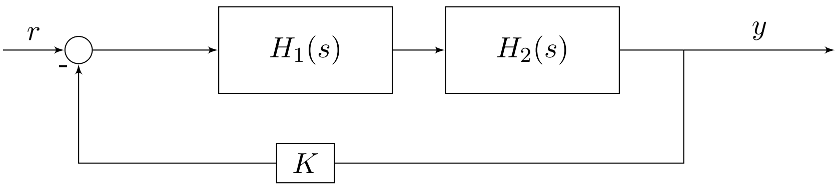


Figure 10. PID controller with $K_p = 350$, $K_d = 50$, $K_i = 300$

Source: http://www.engin.umich.edu/class/ctms/pid/pid.htm

3 Further (typically midterm) problems

1. It is given the following block diagram:



   (a) $H_1(s) = \frac{s+2}{s^2+5s+6}$, $H_2(s) = \frac{1}{s+1}$, $K = 1$, compute the closed-loop (resultant) transfer function $G(s)$! (2p)

   (b) $H_1(s) = \frac{s+1}{s-3}$, $H_2(s) = \frac{s+4}{s^2+3s+2}$. Find K such that the closed-loop is (BIBO) stable? (3p)

   (c) $H_1(s) = \frac{s+2}{s^2+5s+6}$, $H_2(s) =?$, $K = 1$, compute $H_2(s)$ such that, the resultant transfer function has only unstable poles! Adja meg $H_2(s)$-t, úgy hogy csak -tetszőleges- instabil pólusai legyenek az eredő rendszernek (5p)

2. Consider the following transfer function

$$H(s) = \frac{s + l_1}{s^3 + l_2 s^2 + s + 3},$$

   where $l_1 > 0$ and $l_2 < 0$ are real parameters. Does there exist such a finite gain output feedback ($u = -ky$, $k \leq \infty$), which can asymptotically stabilize the system? Létezik-e olyan véges erősítésű lineáris kimenet-visszacsatolás (azaz $u = -ky$, ahol $|k| < \infty$), amely aszimptotikusan stabilizálja a rendszert? Miért? (3p)

3. Compute the gain in decibels of the following transfer function model if the input is a constant function? (2p)

$$H(s) = \frac{s + 1}{s^2 + 10s + 10}$$

4. Decide whether the following transfer function model is minimum phase or not. Why? (2p)

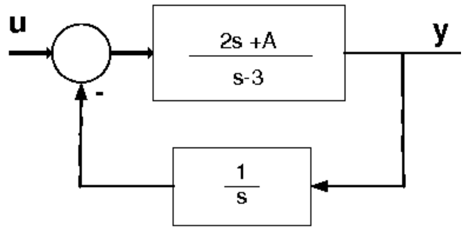
$$H(s) = \frac{(s+1)(s+3)}{s^3 - 3s^2 + 2s + 1}$$

5. It is given the following LTI state-space model $\dot{x} = Ax + Bu$, $y = Cx + Du$, where

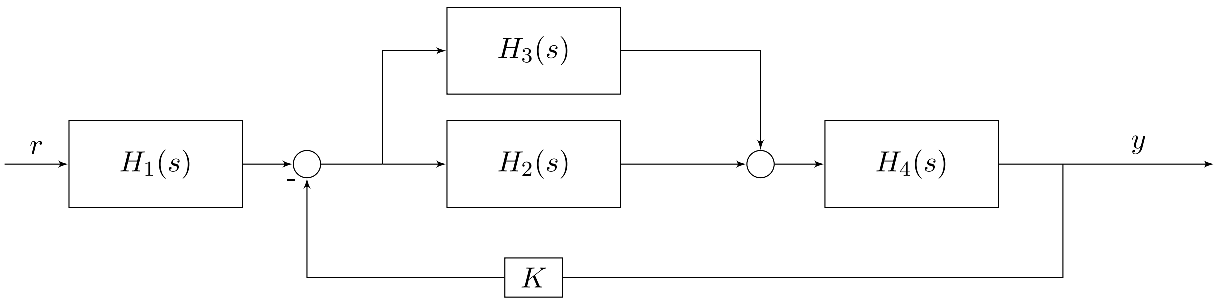
$$A = \begin{bmatrix} 4 & 3.5 \\ 2 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

   (a) Compute the system's transfer function $H(s)$! (3 pont)
   (b) Compute the poles of the system! (1 pont)
   (c) Decide whether the system is stable! Justify your answer! (1 pont)

6. Decide whether the following model is stable or not for $A = 0$ or $A = 0.25$ (3p)?



7. Consider the following transfer function model:



Compute the resulting transfer function $G(s)$ if $H_1(s) = \frac{s+2}{s^2-7s+11}$, $H_2(s) = \frac{1}{s}$, $H_3(s) = \frac{s-3}{s+7}$, $H_4(s) = \frac{s+7}{s+1}$ (6 pont)