

Computer Controlled Systems

Lecture 7

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- 1 Introduction into the control of (SISO) systems
- 2 PID-control

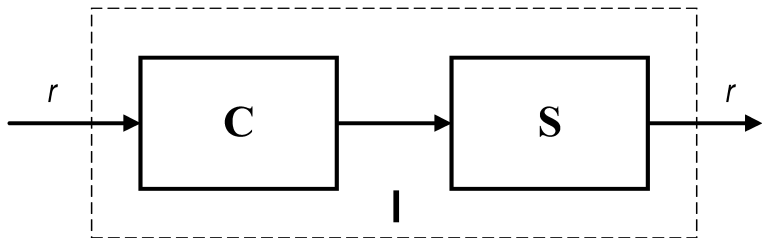
1 Introduction into the control of (SISO) systems

2 PID-control

The control goal

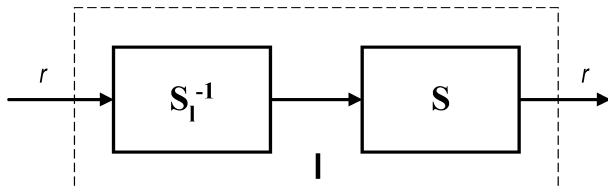
Goal: the output of the system is *identical* to the prescribed reference signal. ("Everything is under control")

Straightforward approach: Let us transform the system operator into the identity operator (the output is exactly the same as the input)

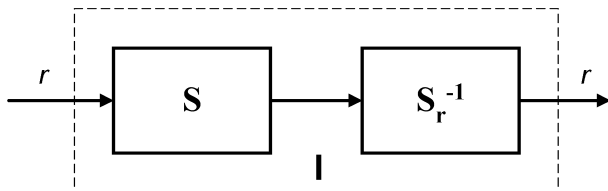


Left and right inverse (MIMO case)

Left inverse:



Right inverse:



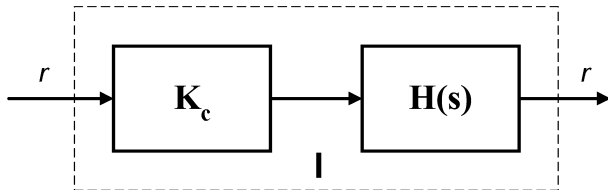
Inversion problems

- The system operator is not invertible
- The system to be controlled is unstable
- The inverse is unstable
- The inverse is not causal (not computable)
- The system operator is uncertain \rightarrow the inverse (might be) even more uncertain
- The system is not isolated in reality (there are external disturbances)

Setting the steady state gain

Assumption: a *stable* SISO transfer function is given

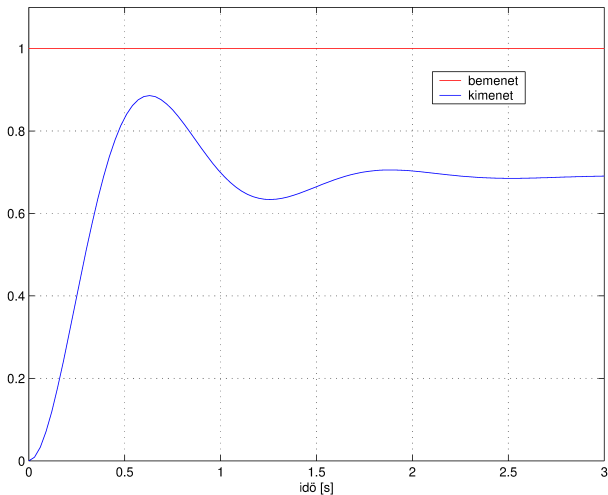
Goal: the output of the "controlled" system should asymptotically follow the constant reference signal (the gain should be 1 at frequency 0)



$$|H(j \cdot 0)| = k \Rightarrow K_c = 1/k$$

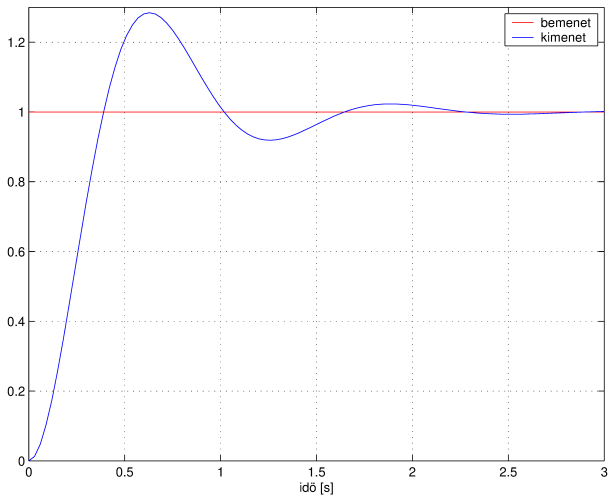
Example – 1

$$H(s) = \frac{20}{s^2 + 4s + 29}, \quad |H(0)| = \frac{20}{29}$$



Example – 2

$$K_c = \frac{29}{20}$$

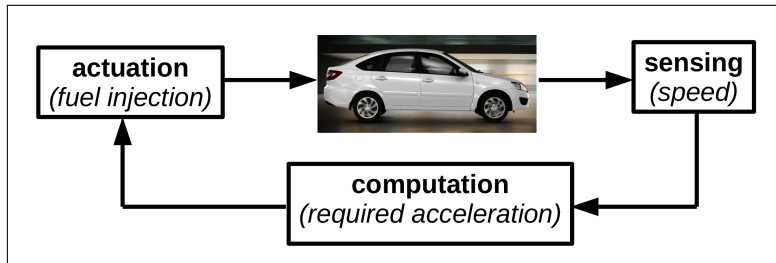


Feedback – 1

Feedback control:

control goal + sensing + feedback computation + actuation

Example: tracking a (constant) speed reference



may fundamentally change the behaviour (dynamical properties) of the original system

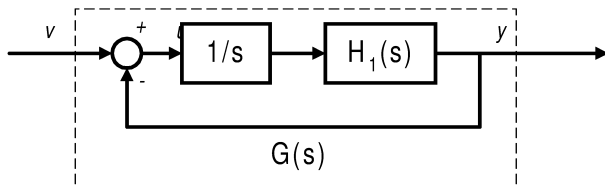
Why to apply?

- Often feedback is the only way to stabilize unstable systems
- A well-designed feedback might operate well even with an uncertain system model
- The effect of external disturbances can also be reduced by feedback

Types of feedback

- *output feedback*: the input only depends on the outputs of the system, i.e. $u = \mathbf{F}[y]$
- *(full) state feedback*: the input depends on the state variables of the system, i.e. $u = \mathbf{F}[x]$
- *static feedback*: the \mathbf{F} operator is static ($u = F(y)$, $u = F(x)$)
- *dynamic feedback*: the \mathbf{F} operator is dynamic (can be given by a state space model or a transfer function)
- *linear feedback*: the \mathbf{F} operator or the F function is linear

Role of the integrator



$$H_1(s) = \frac{b(s)}{a(s)} \Rightarrow G(s) = \frac{k_I \cdot b(s)}{s \cdot a(s) + k_I \cdot b(s)}$$

$$|G(j \cdot 0)| = 1$$

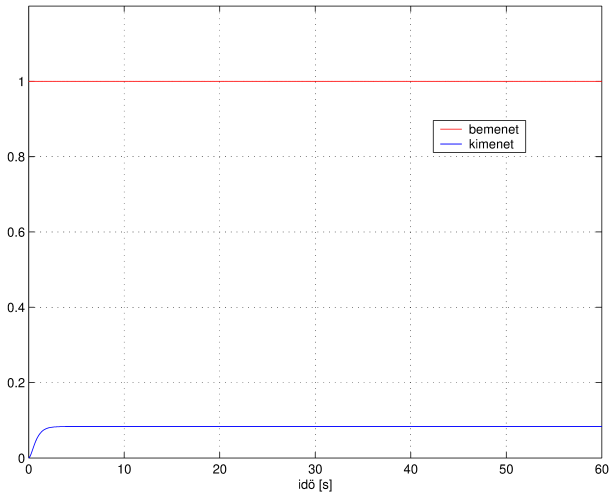
The steady state gain of a stable controlled system containing an integrator is 1.

(The controlled system follows the constant reference signal, if it is asymptotically stable.)

Example – 1

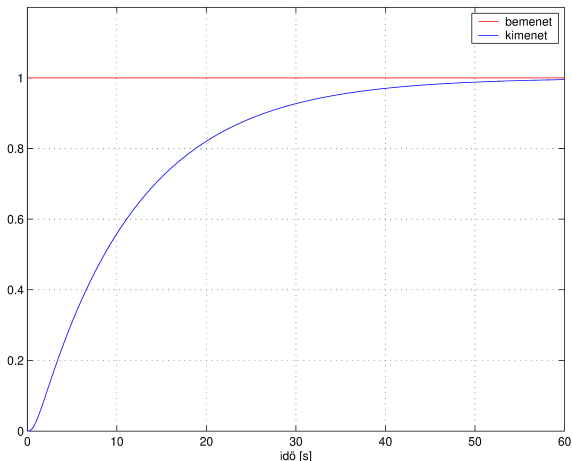
System model: $H(s) = \frac{0.5}{s^2 + 5s + 6}$

Response for a unit step input:



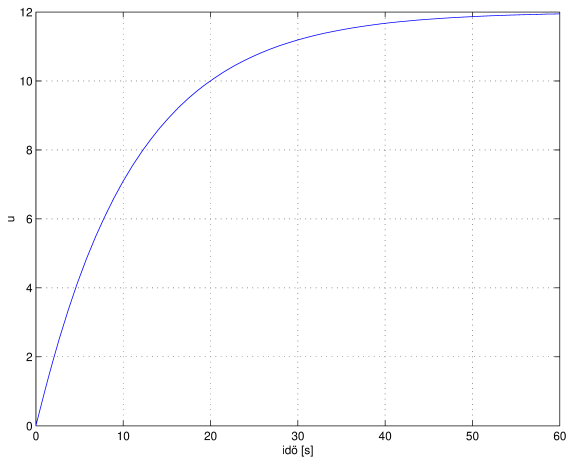
Example – 2

Controlled system containing an integrator ($k_I = 1$): $G(s) = \frac{0.5}{s^3 + 5s^2 + 6s + 0.5}$
Response to a unit step input:



Example – 3

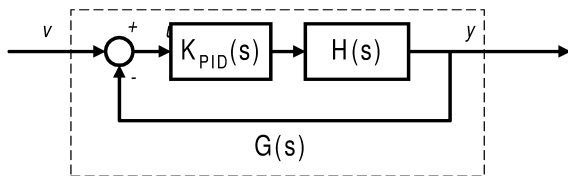
output of the integrator \equiv input of the original system:



1 Introduction into the control of (SISO) systems

2 PID-control

Structure of PID controllers – 1

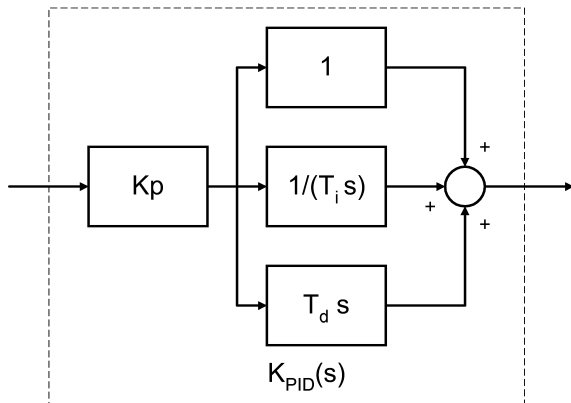


P=Proportional, **I**=Integral, **D**=Derivative

Transfer function:

$$K_{PID}(s) = K_p \left[1 + \frac{1}{T_i \cdot s} + T_d \cdot s \right] = \frac{K_p (T_i \cdot T_d \cdot s^2 + T_i \cdot s + 1)}{T_i \cdot s}$$

Structure of PID controllers – 2



K_p : proportional gain

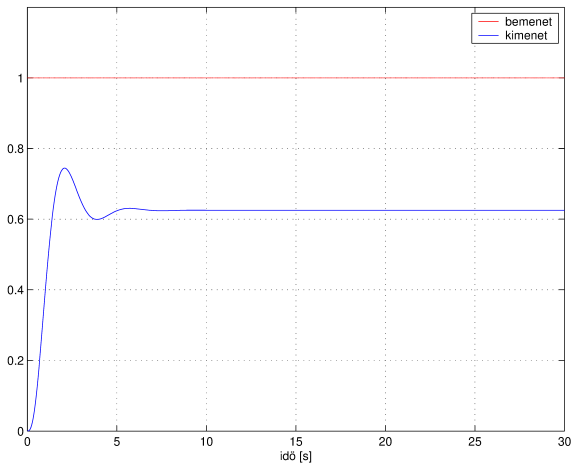
T_i : integration time constant

T_d : derivation time constant

PID design example – 1

System model: $H(s) = \frac{10}{s^3 + 6s^2 + 11s + 16}$

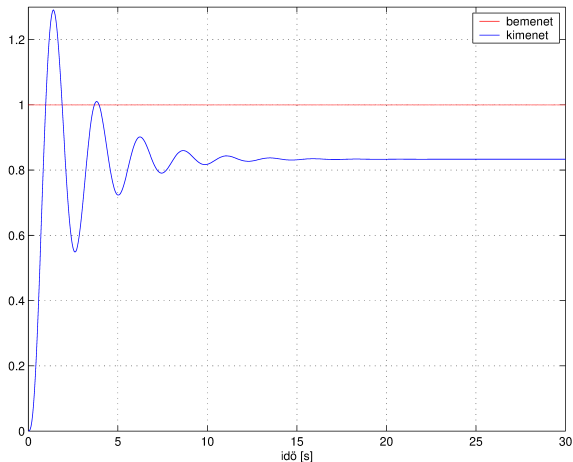
Step response



PID design example – 2

Proportional (P) feedback: $K_p = 3$, $G(s) = \frac{30}{s^3 + 6s^2 + 11s + 36}$

Unit step response

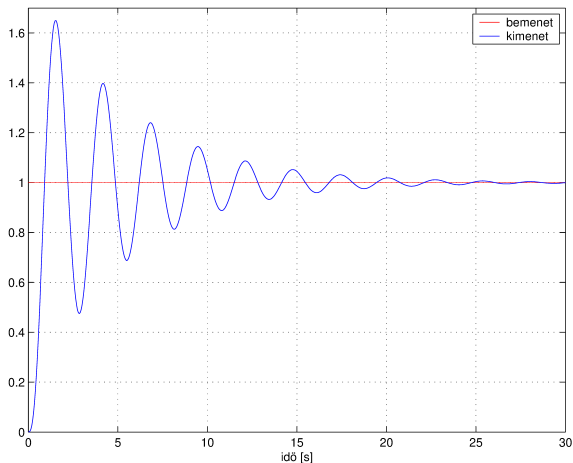


PID design example – 3

Proportional + integrator (PI) feedback: $K_p = 2.7$, $T_i = 1.5$,

$$G(s) = \frac{40.5s+27}{1.5s^4+9s^3+16.5s^2+49.5s+27}$$

Unit step response

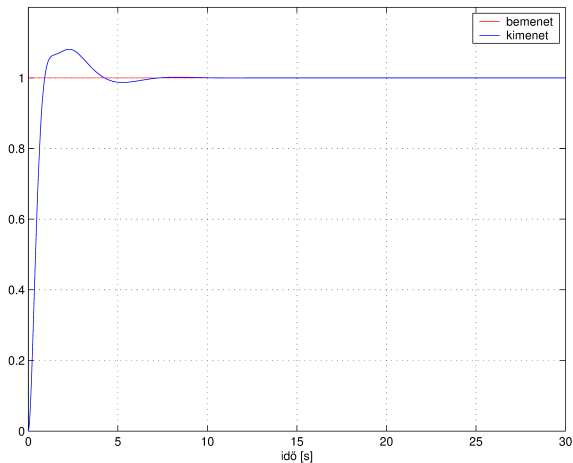


PID design example – 4

Proportional + integrator + derivator (PID) feedback: $K_p = 2$, $T_i = 0.9$,

$$T_d = 0.6, G(s) = \frac{10.8s^2 + 18s + 20}{0.9s^4 + 5.4s^3 + 20.7s^2 + 23.4s + 20}$$

Unit step response



Tuning of PID controllers – 1

Ziegler-Nichols method

- 1 Apply a simple proportional feedback
- 2 Increase the proportional gain (K_p) until the step response becomes an undamped (sinusoidal) oscillation. The critical gain is K_p^* .
- 3 Measure the period of the oscillation (T_c)

Tuning of PID controllers – 2

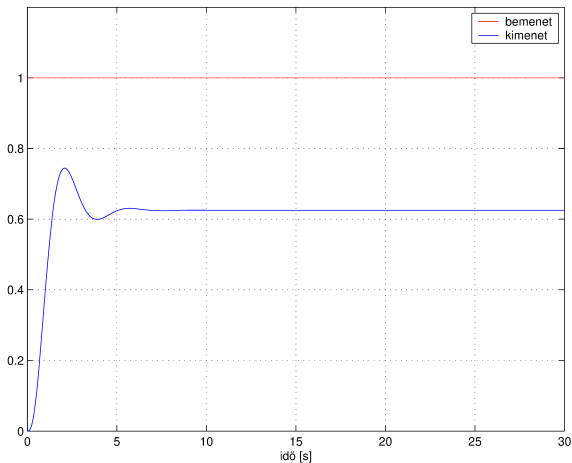
Controller tuning:

- **P-controller:** $K_p = 0.5K_p^*$
- **PI-controller:** $K_p = 0.45K_p^*$, $T_i = 0.833T_c$
- **PID-controller (fast):** $K_p = 0.6K_p^*$, $T_i = 0.5T_c$, $T_d = 0.125T_c$
- **PID-controller (small overshoot):** $K_p = 0.33K_p^*$, $T_i = 0.5T_c$,
 $T_d = 0.33T_c$
- **PID-controller (without overshoot):** $K_p = 0.2K_p^*$, $T_i = 0.3T_c$,
 $T_d = 0.5T_c$

Example – 1

System model: $H(s) = \frac{40}{2s^3 + 10s^2 + 82s + 10}$

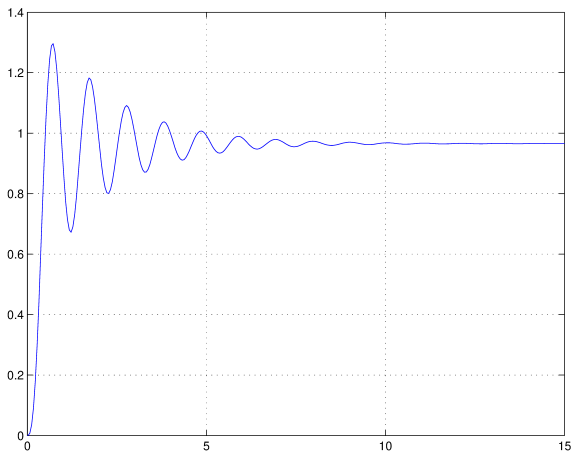
Step response:



Example – 2

Proportional feedback, $K_p = 7$

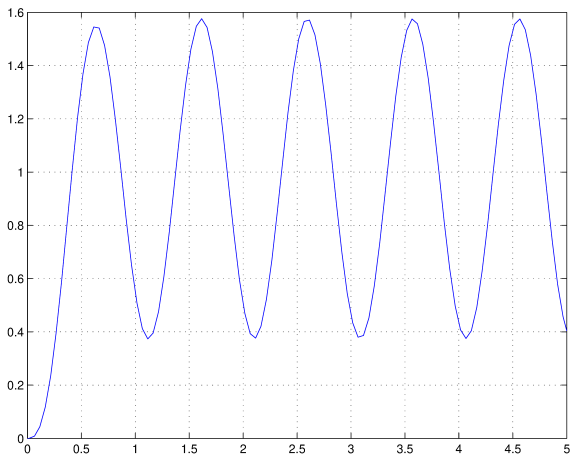
Step response:



Example – 3

Proportional feedback, $K_p^* = 10$, $T_c = 1$

Step response:



Example – 4

PID controller parameters: $K_p = 3.3$, $T_i = 0.5$, $T_d = 0.33$

Controller transfer function:

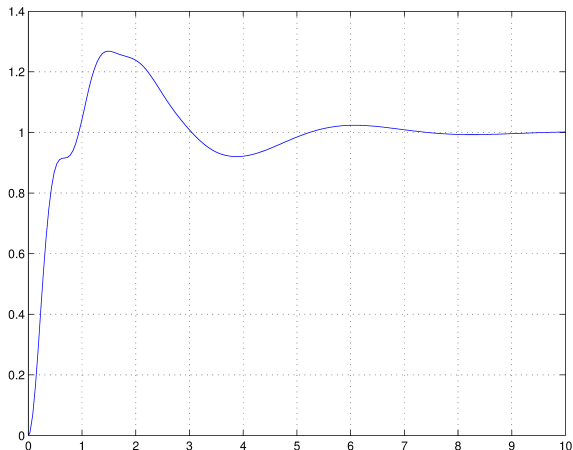
$$K_{PID}(s) = \frac{K_p(T_i \cdot T_d \cdot s^2 + T_i \cdot s + 1)}{T_i \cdot s}$$

Transfer function of the closed loop system:

$$G(s) = \frac{21.78s^2 + 66s + 132}{s^4 + 5s^3 + 62.78s^2 + 71s + 132}$$

Example – 5

Step response of the controlled system



Example: DC motor – 1

System equations, parameters and variables:

| | | |
|-----|----------------------------------|----------------------------------|
| J | moment of inertia | $0.01 \text{ kg m}^2/\text{s}^2$ |
| b | damping coefficient | 0.1 Nm s |
| K | electromotive torque coefficient | 0.01 Nm/A |
| R | resistance | 1 ohm |
| L | inductance | 0.5 H |

state variables, input, output:

$x_1 = \dot{\theta}$ angular velocity [rad/s]

$x_2 = i$ current [A]

u input voltage [V]

$y = x_1$

Example: DC motor – 2

State space model:

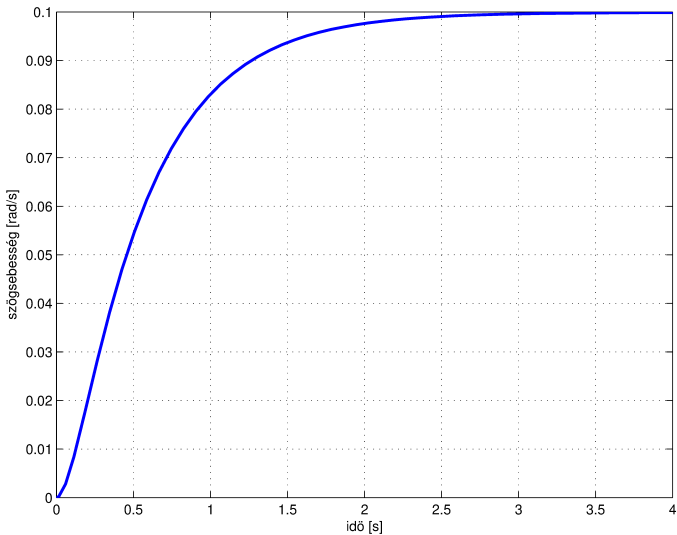
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Transfer function:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{K}{(Js + b)(Ls + R) + K^2}$$

Example: DC motor – 3

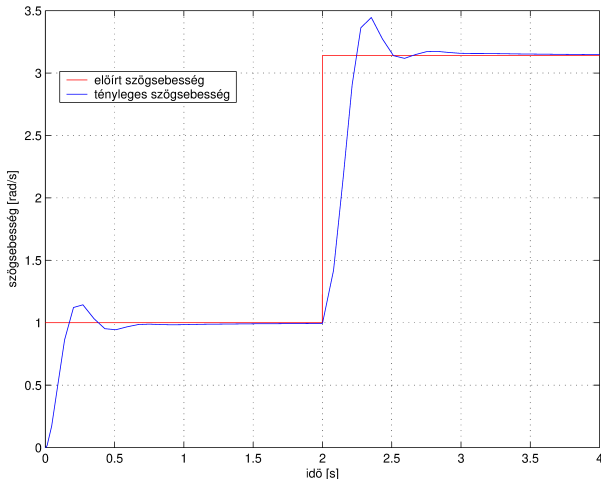
Response to $u = 1V$ input:



Example: DC motor – 4

PID-parameters: $K_p = 100$, $T_i = 1/100$, $K_d = 1$

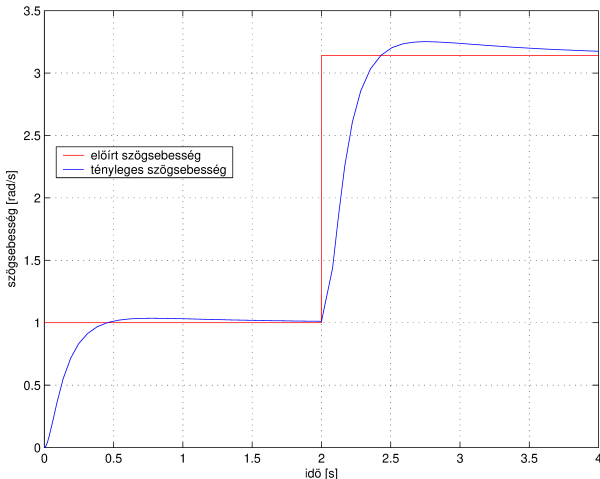
Operation of the controlled system:



Example: DC motor – 5

PID-parameters: $K_p = 100$, $T_i = 1/100$, $K_d = 10$

Operation of the controlled system:



Evaluation of SISO control systems – 1

Time domain, unit step response

- e_{max} : maximal overshoot
- t_{max} : time of maximal overshoot
- T_a ($T_{a,50}$): rise time
- T_u : delay
- t_ϵ : settling time

Evaluation of SISO control systems – 2

Time domain, measuring the difference from the reference

- $I_1 = \int_0^{\infty} e(t) dt$
- $I_2 = \int_0^{\infty} |e(t)| dt$
- $I_3 = \int_0^{\infty} e^2(t) dt$
- $I_4 = \int_0^{\infty} [e^2(t) + \alpha \dot{e}^2(t)] dt$
- $I_5 = \int_0^{\infty} [e^2(t) + \beta u^2(t)] dt$

Summary

- typical control goals: output reference following (tracking), stabilization, disturbance rejection
- inversion: important theoretical concept, typically not directly implementable
- feedback: helps to achieve several control goals
- classification of feedback types is important
- static output feedback is often not enough even for stabilization
- PID control: frequently used dynamic output feedback with only 3 parameters
- evaluation criteria: help in comparison, acceptance decision