Computer Controlled Systems

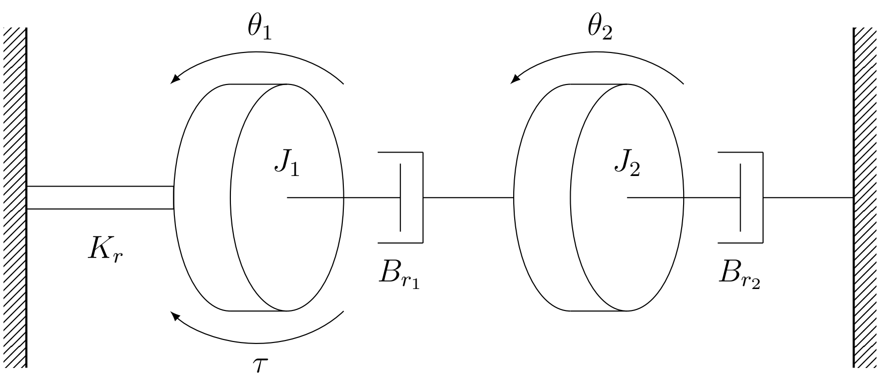
Homework 2.

Submission deadline: 19th of November, at 10:00 (approx. 4 weeks)

All solutions are expected to be calculated by hand, also all figures have to be drawn by hand. Computer programs (e.g. Matlab) can be used for self-verification, but all problems have to contain the detailed steps of solutions

Problems

1. Let us consider the following rotating system with two flywheels:



The first flywheel has moment of inertia J_1 and is attached by a flexible shaft with spring constant K_r to the wall. We apply an external torque τ (which is the input). The second flywheel has moment of inertia J_2 is driven by friction between the two flywheels with friction coefficient B_{r_1} (which will be modeled with a damper). The second flywheel also has friction to the other wall with friction coefficient B_{r_2}. The output of the system is the first flywheel's angle, θ_1.

Using D'Alembert's Law we know that the sum of all torques (including the initial torque) should be zero, i.e. for the first flywheel we can write the following equation:

$$\tau + J_1\ddot{\theta}_1 + K_r\theta_1 + B_{r_1}\big(\dot{\theta}_1 - \dot{\theta}_2\big) = 0.$$

Similarly for the second flywheel:

$$J_2\ddot{\theta}_2 + B_{r_2}\dot{\theta}_2 - B_{r_1}\big(\dot{\theta}_1 - \dot{\theta}_2\big) = 0.$$

The above equations can be rearranged in the form:

$$\ddot{\theta}_1 = \frac{1}{J_1}\big(-\tau - K_r\theta_1 - B_{r_1}\dot{\theta}_1 + B_{r_1}\dot{\theta}_2\big)$$
$$\ddot{\theta}_2 = \frac{1}{J_2}\big(-(B_{r_1} + B_{r_2})\dot{\theta}_2 + B_{r_1}\dot{\theta}_2\big).$$

Using the state variables, input and output functions

$$x_1 = \theta_1$$
$$x_2 = \dot{\theta}_1$$
$$x_3 = \dot{\theta}_2$$
$$u = \tau$$
$$y = x_1$$

we can rewrite the equations describing the dynamics of the system as

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{J_1}\left(-u - K_r x_1 - B_{r_1} x_2 + B_{r_1} x_3\right) \\
\dot{x}_3 &= \frac{1}{J_2}\left(-(B_{r_1} + B_{r_2}) x_3 + B_{r_1} x_2\right) \\
y &= x_1
\end{aligned}$$

and in matrix form

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ -\frac{K_r}{J_1} & -\frac{B_{r1}}{J_1} & \frac{B_{r1}}{J_1} \\ 0 & \frac{B_{r1}}{J_2} & -\frac{B_{r1}+B_{r2}}{J_2} \end{bmatrix} x + \begin{bmatrix} 0 \\ -\frac{1}{J_1} \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x.
\end{aligned}$$

(a) Substitute the following numbers and rewrite the state space representation:

$$K_r = 1 \quad B_{r_1} = 1 \quad B_{r_2} = 4 \quad J_1 = 0.2 \quad J_2 = 2.$$

(b) Is the system exponentially stable?

(c) Explain why the impulse response of the system is oscillating for a while!

(d) Design a static state feedback controller such that the closed loop system will not oscillate at all. For example, place the poles of the close loop system into -1, -2 and -3. *The numerical computations are difficult, you are allowed to use Matlab, but please, document each subresult (e.g. $\mathcal{C}_3$, $\mathcal{C}_3^{-1}$, T_l^{-1},...).*

(e) Design an observer gain L such that the error dynamics of the Luenberger observer tends exponentially to zero. *You are allowed again to use the results of the Matlab computations.*

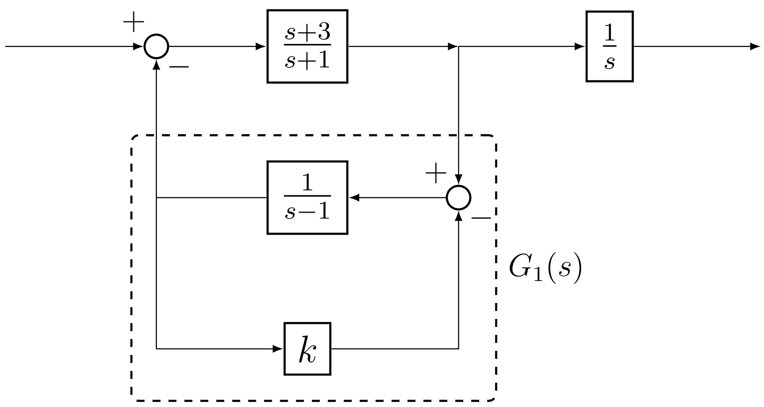
Compulsory only for TP students, ***but extra points for others***.

(f) The energy stored in a flexible shaft (rotational spring) and a rotating mass is

$$\begin{aligned}
E_K &= \frac{1}{2} K \theta^2 \\
E_J &= \frac{1}{2} J \dot{\theta}^2.
\end{aligned}$$

Check that the total energy function $V = E_{K_r} + E_{J_1} + E_{J_2}$ is an appropriate Lyapunov function.

2. Let us consider the following block diagram:



(a) Compute the resulting transfer function $G(s)$. It is advised to first determine the resulting transfer function $G_1(s)$ of the highlighted subsystem.

(b) Choose the value of k such that resulting transfer function is stable.