Computer Controlled Systems

Homework 1. (revised version)

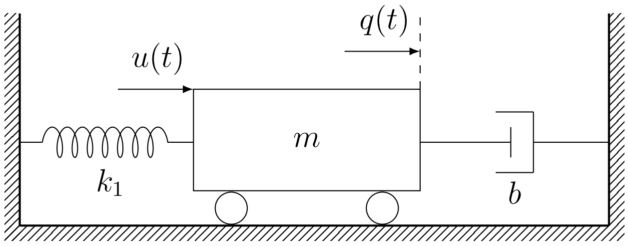
Extended submission deadline: ~~15th of October~~ **18th of October**, at 12:00 (approx. 3 weeks)

The modifications are indicated with blue color.

All solutions are expected to be calculated by hand, also all figures have to be drawn by hand. Computer programs (e.g. Matlab) can be used for self-verification, but all problems have to contain the detailed steps of solutions

Problems

1. Let us consider a cart with mass m connected to a spring with spring coefficient k_1 and a damper with damping coefficient b. Let $q(t)$ describe the displacement of the cart. We also consider an external force applied to the car, $u(t)$.



Using Newton's second law, we can we can write:

$$m\ddot{q}(t) = -b\dot{q}(t) - k_1 q(t) + u(t)$$

and rearranging the equation we get the following differential equation, which describes the cart's position:

$$m\ddot{q}(t) + b\dot{q}(t) + k_1 q(t) = u(t).$$

Using the values $m = 1$, $k_1 = 5$ and $b = 6$ and initial values $\dot{q}(0) = -1$ and $q(0) = 1$, compute $q(t)$ with Laplace transformation, if the input is:

(a) $u(t) = 4e^{-t}$

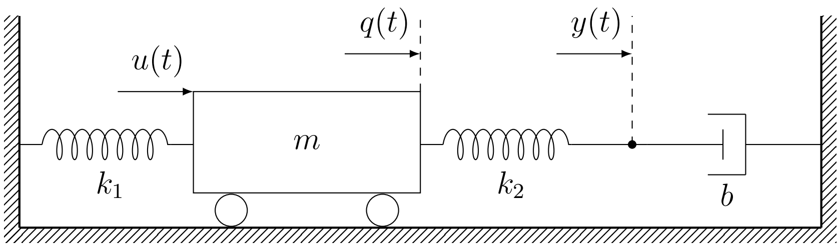
Compulsory only for TP students, ***but extra points for others***. It is advised to help your computations with Matlab and/or other symbolic math software tool.

(b) $u(t) = e^{-2t}$

(c) $u(t) = e^{-5t}\cos 3t$

(d) $u(t) = 12e^{-3t}\sinh(2t)$.

2. Let us consider the previous example with an additional spring between the cart and the damper, with spring coefficient k_2. We measure the displacement of the point between the spring and the damper, i.e. the output of the system is $y(t)$.



Using the same methods as before, we can write the following equations:

$$m\ddot{q}(t) = -k_1 q(t) + k_2\big(y(t) - q(t)\big) - u(t)$$
$$b\dot{y}(t) = -k_2\big(y(t) - q(t)\big).$$

Using the state variables

$$x_1(t) = q(t)$$
$$x_2(t) = \dot{q}(t)$$
$$x_3(t) = y(t)$$

we can rewrite the above equations as:

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -\frac{k_1 + k_2}{m} x_1(t) + \frac{k_2}{m} x_3(t) - \frac{1}{m} u(t)$$
$$\dot{x}_3(t) = \frac{k_2}{b} x_1(t) - \frac{k_2}{b} x_3(t).$$

The state space representation of the system is:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k_1+k_2}{m} & 0 & \frac{k_2}{m} \\ \frac{k_2}{b} & 0 & -\frac{k_2}{b} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

(a) Substitute the following values to the state space representation!

$$k_1 = 1,\ k_2 = 10,\ b = \frac{5}{3},\ m = 1$$

(b) Compute the eigenvalues and the corresponding eigenvectors of matrix A!

(c) Determine the diagonal matrix D and give an appropriate coordinate transformation S such that $D = S^{-1}AS$!

(d) Compute the exponential matrix e^D!

(e) Compute the exponential matrix e^A!

(f) Compute the determinant of matrix A!

(g) Compute the kernel space and the image space of matrix A!

(h) Compute the transfer function $H(s)$ of the system using Laplace transformation!

(i) Compute the impulse-response function $h(t)$ of the system!

Compulsory only for TP students, ***but extra points for others***.

3. Let us consider the previous example with an additional spring with spring coefficient k_3. Based on the previous exercises derive the differential equations describing the displacement of the cart and the point between the springs! Transform the equation into the form $\dot{x}(t) = Ax(t) + Bu(t)$.