Computer Controlled Systems
Lecture 7

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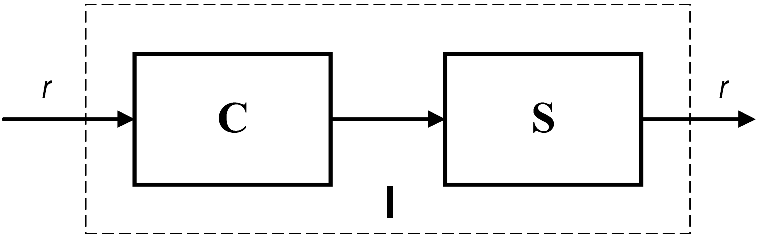
1. Introduction into the control of (SISO) systems

2. PID-control

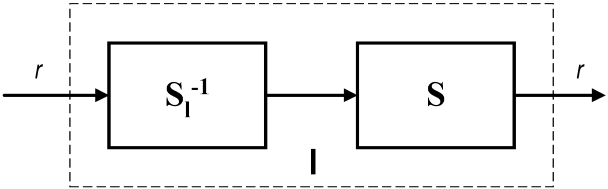
The control goal

Goal: the output of the system is *identical* to the prescribed reference signal. ("Everything is under control")
Straightforward approach: Let us transform the system operator into the identity operator (the output is exactly the same as the input)

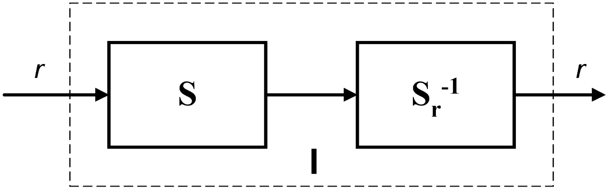


Left and right inverse (MIMO case)

Left inverse:



Right inverse:

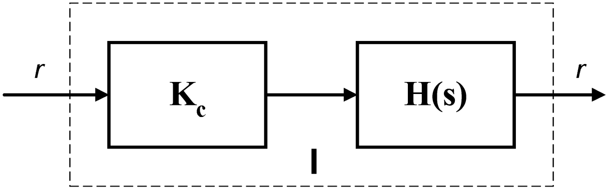


Inversion problems

- The system operator is not invertible
- The system to be controlled is unstable
- The inverse is unstable
- The inverse is not causal (not computable)
- The system operator is uncertain $\rightarrow$ the inverse (might be) even more uncertain
- The system is not isolated in reality (there are external disturbances)

Setting the steady state gain

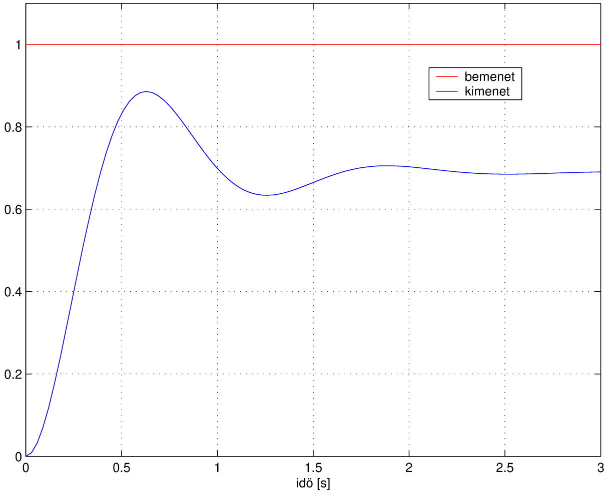
Assumption: a *stable* SISO transfer function is given
Goal: the output of the "controlled" system should asymptotically follow the constant reference signal (the gain should be 1 at frequency 0)



$|H(j \cdot 0)| = k \Rightarrow K_c = 1/k$

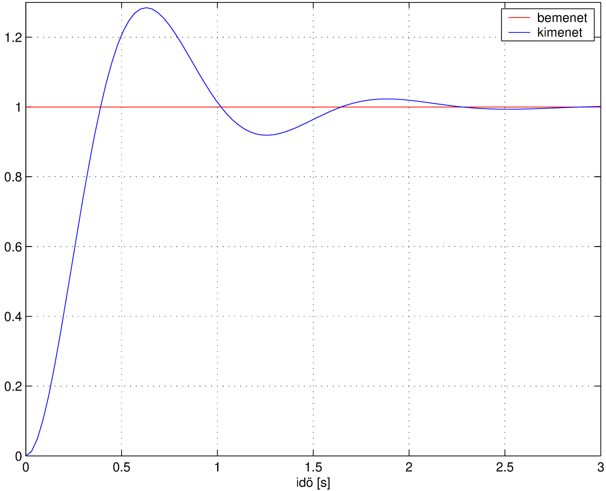
Example – 1

$H(s) = \frac{20}{s^2+4s+29}$, $|H(0)| = \frac{20}{29}$



Example – 2

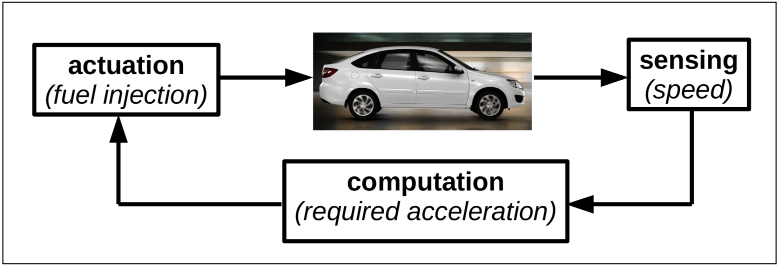
$K_c = \frac{29}{20}$



Feedback – 1

Feedback control:
control goal + sensing + feedback computation + actuation

Example: tracking a (constant) speed reference



may fundamentally change the behaviour (dynamical properties) of the original system

Feedback – 2

Why to apply?

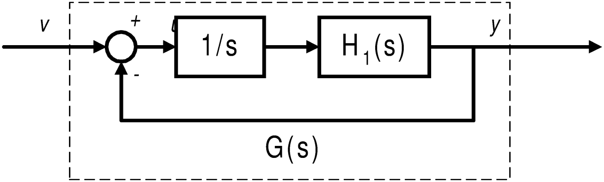
- Often feedback is the only way to stabilize unstable systems
- A well-designed feedback might operate well even with an uncertain system model
- The effect of external disturbances can also be reduced by feedback

Feedback – 3

Types of feedback

- *output feedback*: the input only depends on the outputs of the system, i.e. $u = \mathbf{F}[y]$
- *(full) state feedback*: the input depends on the state variables of the system, i.e. $u = \mathbf{F}[x]$
- *static feedback*: the $\mathbf{F}$ operator is static ($u = F(y)$, $u = F(x)$)
- *dynamic feedback*: the $\mathbf{F}$ operator is dynamic (can be given by a state space model or a transfer function)
- *linear feedback*: the $\mathbf{F}$ operator or the F function is linear

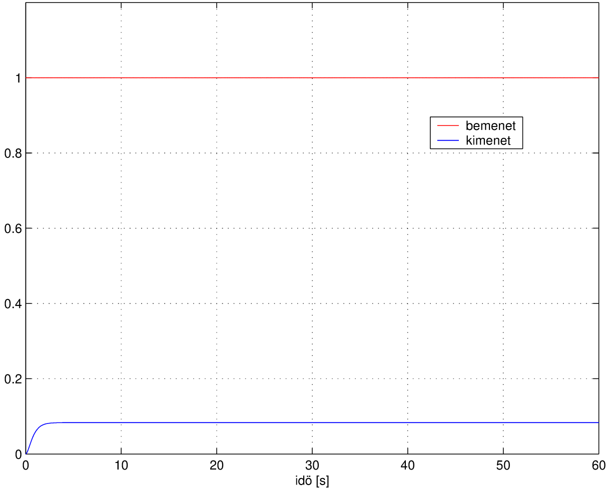
Role of the integrator



$H_1(s) = \frac{b(s)}{a(s)} \Rightarrow G(s) = \frac{k_I \cdot b(s)}{s \cdot a(s) + k_I \cdot b(s)}$
$|G(j \cdot 0)| = 1$
The steady state gain of a stable controlled system containing an integrator is 1.
(The controlled system follows the constant reference signal, if it is asymptotically stable.)

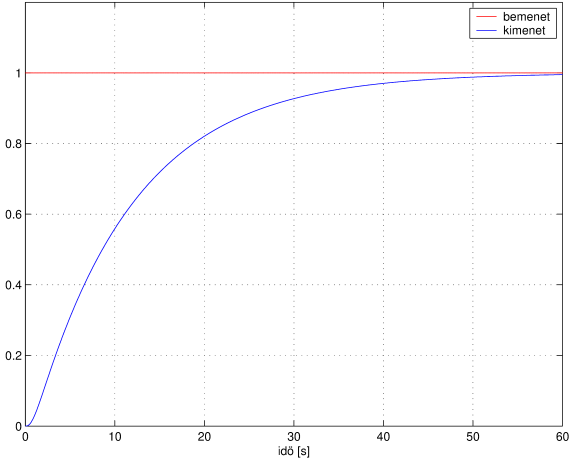
Example – 1

System model: $H(s) = \frac{0.5}{s^2+5s+6}$
Response for a unit step input:



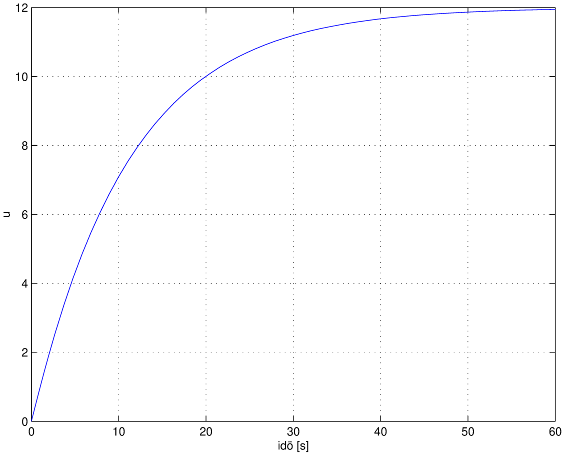
Example – 2

Controlled system containing an integrator ($k_I = 1$): $G(s) = \frac{0.5}{s^3+5s^2+6s+0.5}$
Response to a unit step input:

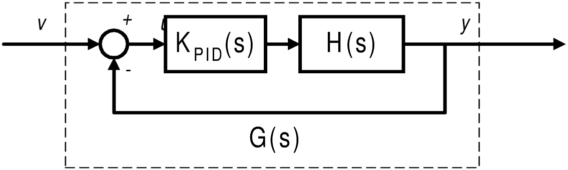


Example – 3

output of the integrator ≡ input of the original system:



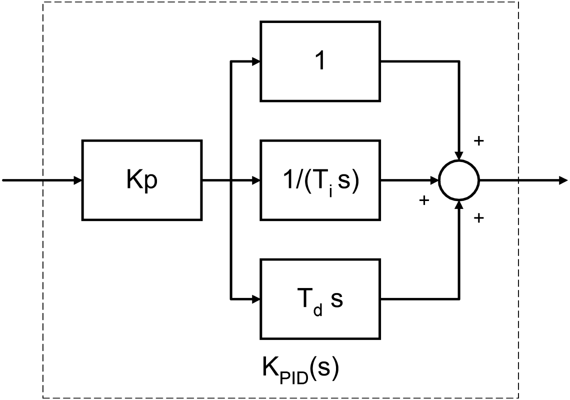
Structure of PID controllers – 1



P=Proportional, **I**=Integral, **D**=Derivative
Transfer function:

$$K_{PID}(s) = K_p \left[1 + \frac{1}{T_i \cdot s} + T_d \cdot s \right] = \frac{K_p(T_i \cdot T_d \cdot s^2 + T_i \cdot s + 1)}{T_i \cdot s}$$

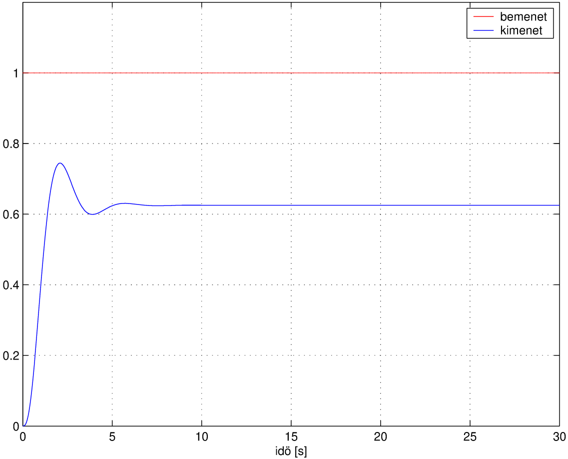
Structure of PID controllers – 2



K_p: proportional gain
T_i: integration time constant
T_d: derivation time constant

PID design example – 1

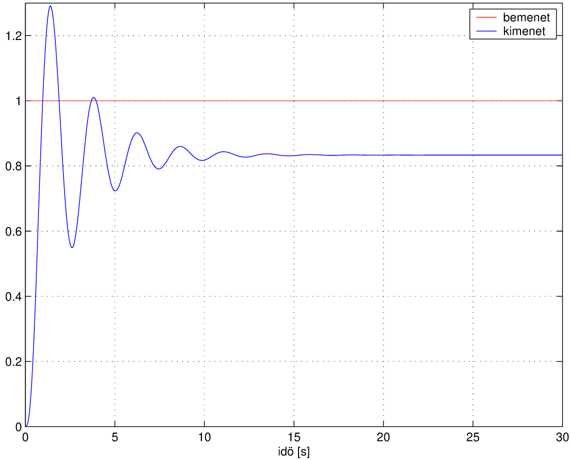
System model: $H(s) = \frac{10}{s^3+6s^2+11s+16}$
Step response



PID design example – 2

Proportional (P) feedback: $K_p = 3$, $G(s) = \frac{30}{s^3+6s^2+11s+36}$

Unit step response

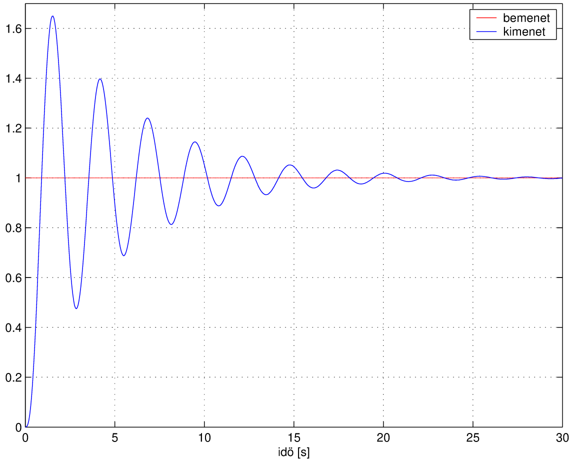


PID design example – 3

Proportional + integrator (PI) feedback: $K_p = 2.7$, $T_i = 1.5$,

$G(s) = \frac{40.5s+27}{1.5s^4+9s^3+16.5s^2+49.5s+27}$

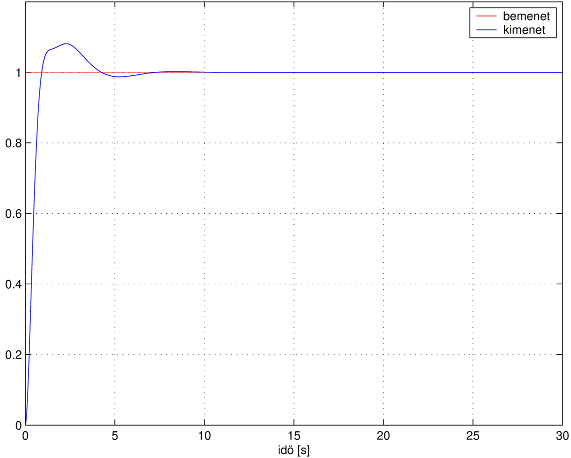
Unit step response



PID design example – 4

Proportional + integrator + derivator (PID) feedback: $K_p = 2$, $T_i = 0.9$, $T_d = 0.6$, $G(s) = \frac{10.8s^2+18s+20}{0.9s^4+5.4s^3+20.7s^2+23.4s+20}$

Unit step response



Tuning of PID controllers – 1

Ziegler-Nichols method

1. Apply a simple proportional feedback
2. Increase the proportional gain (K_p) until the step response becomes an undamped (sinusoidal) oscillation. The critical gain is K_p^*.
3. Measure the period of the oscillation (T_c)

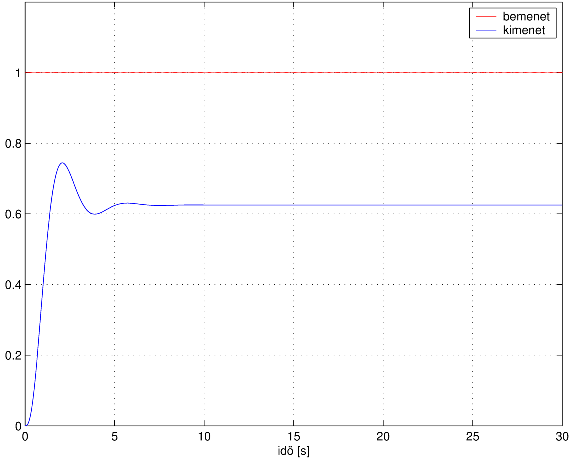
Tuning of PID controllers – 2

Controller tuning:

- **P-controller**: $K_p = 0.5K_p^*$
- **PI-controller**: $K_p = 0.45K_p^*,\ T_i = 0.833T_c$
- **PID-controller (fast)**: $K_p = 0.6K_p^*,\ T_i = 0.5T_c,\ T_d = 0.125T_c$
- **PID-controller (small overshoot)**: $K_p = 0.33K_p^*,\ T_i = 0.5T_c,$ $T_d = 0.33T_c$
- **PID-controller (without overshoot)**: $K_p = 0.2K_p^*,\ T_i = 0.3T_c,$ $T_d = 0.5T_c$

Example – 1

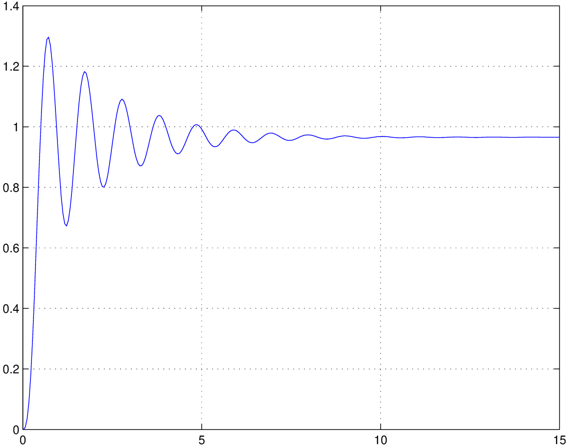
System model: $H(s) = \frac{40}{2s^3+10s^2+82s+10}$
Step response:



Example – 2

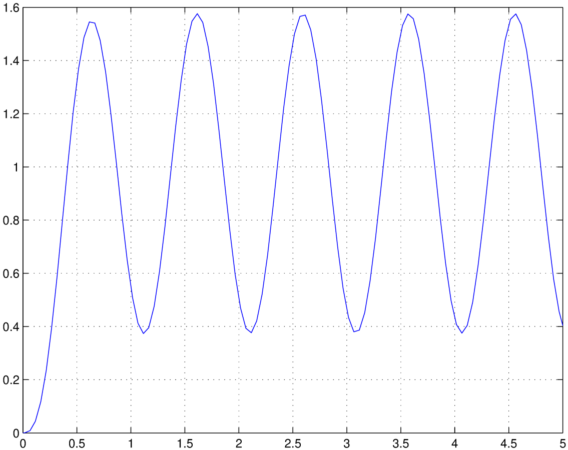
Proportional feedback, $K_p = 7$

Step response:



Example – 3

Proportional feedback, $K_p^* = 10$, $T_c = 1$
Step response:



Example – 4

PID controller parameters: $K_p = 3.3,\ T_i = 0.5,\ T_d = 0.33$
Controller transfer function:

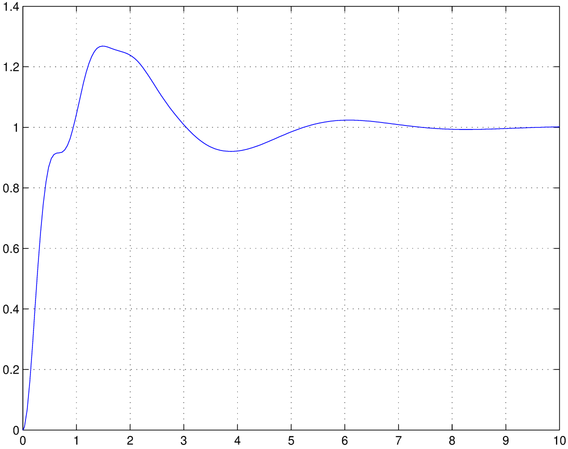
$$K_{PID}(s) = \frac{K_p(T_i \cdot T_d \cdot s^2 + T_i \cdot s + 1)}{T_i \cdot s}$$

Transfer function of the closed loop system:

$$G(s) = \frac{21.78s^2 + 66s + 132}{s4 + 5s^3 + 62.78s^2 + 71s + 132}$$

Example – 5

Step response of the controlled system



Example: DC motor – 1

System equations, parameters and variables:

J	moment of inertia	0.01 kg m^2/s^2
b	damping coefficient	0.1 Nm s
K	electromotive torque coefficient	0.01 Nm/A
R	resistance	1 ohm
L	inductance	0.5 H

state variables, input, output:

$x_1 = \dot{\theta}$ angular velocity [rad/s]

$x_2 = i$ current [A]

u input voltage [V]

$y = x_1$

Example: DC motor – 2

State space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$$

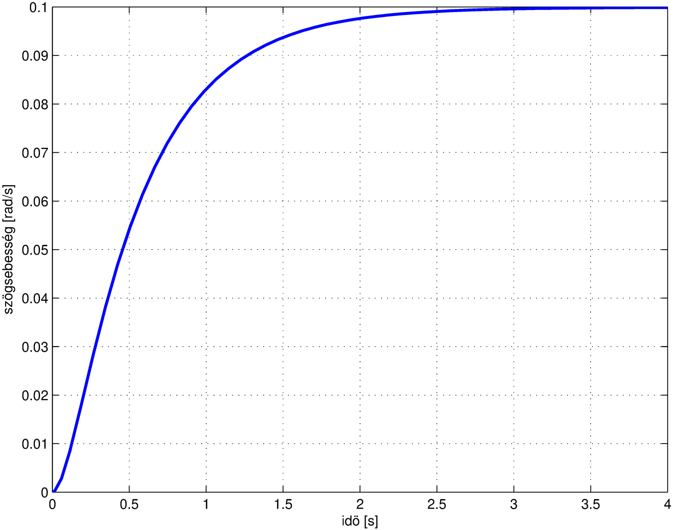
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Transfer function:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{K}{(Js+b)(Ls+R)+K^2}$$

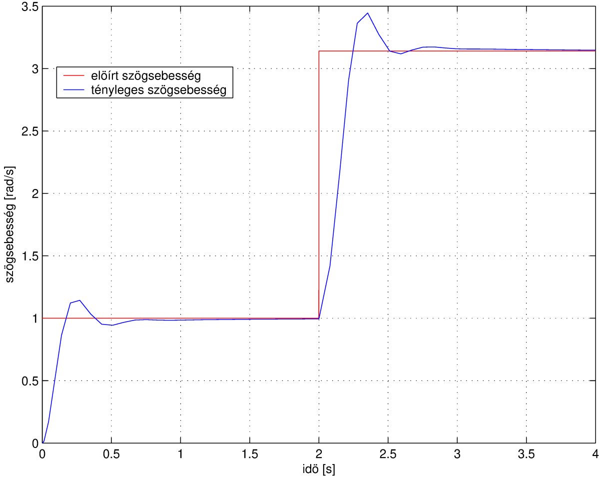
Example: DC motor – 3

Response to $u = 1V$ input:



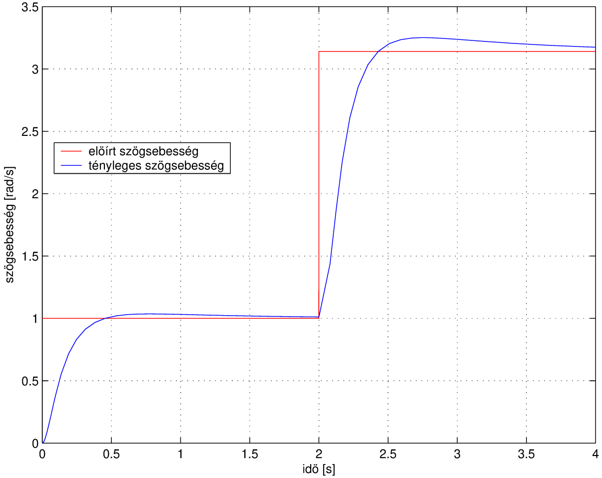
Example: DC motor – 4

PID-parameters: $K_p = 100,\ T_i = 1/100,\ K_d = 1$
Operation of the controlled system:



Example: DC motor – 5

PID-parameters: $K_p = 100$, $T_i = 1/100$, $K_d = 10$
Operation of the controlled system:



Evaluation of SISO control systems – 1

Time domain, unit step response

- e_{max}: maximal overshoot
- t_{max}: time of maximal overshoot
- T_a ($T_{a,50}$): rise time
- T_u: delay
- t_ϵ: settling time

Time domain, measuring the difference from the reference

- $I_1 = \int_0^\infty e(t)dt$
- $I_2 = \int_0^\infty |e(t)|dt$
- $I_3 = \int_0^\infty e^2(t)dt$
- $I_4 = \int_0^\infty [e^2(t) + \alpha\dot{e}^2(t)]dt$
- $I_5 = \int_0^\infty [e^2(t) + \beta u^2(t)]dt$

Summary

- typical control goals: output reference following (tracking), stabilization, disturbance rejection
- inversion: important theoretical concept, typically not directly implementable
- feedback: helps to achieve several control goals
- classification of feedback types is important
- static output feedback is often not enough even for stabilization
- PID control: frequently used dynamic output feedback with only 3 parameters
- evaluation criteria: help in comparison, acceptance decision