Computer Controlled Systems

Homework 3.

Submission deadline: **29th of November**, at 13:00 (approx. 3 weeks)

All solutions are expected to be calculated by hand, also all figures have to be drawn by hand. Computer programs (e.g. Matlab) can be used for self-verification, but all problems have to contain the detailed steps of solutions

Problem I. Dynamics of the electric motor

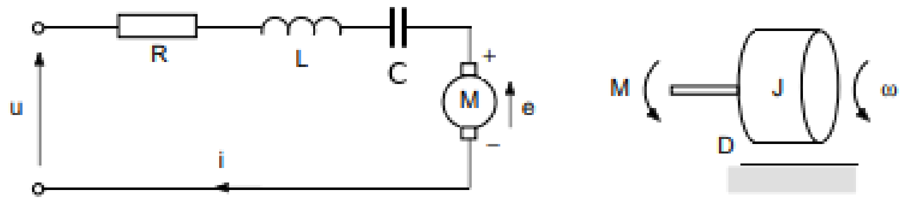


Figure 1: Schematic diagram of an electric motor.

1. Consider a simple electric motor, represented by the above picture. We can describe its operation with use of differential equations. The momentum balance of the rotor can be obtained by Newton's 2nd law for rotation:

$$J\dot{\omega}(t) = \text{External torque} - \text{Friction} \quad \Rightarrow \quad J\dot{\omega}(t) = k\,i(t) - D\,\omega$$
$$\Rightarrow \quad \dot{\omega}(t) = \frac{k\,i(t)}{J} - \frac{D\,\omega}{J}, \tag{1}$$

where $\omega(t)$ is the angular velocity, J is the moment of inertia of the rotor, $i(t)$ denotes the current given to the motor, k is the motor torque constant. The Kirchoff's laws for the electric circuit are the following:

$$v_L(t)+v_C(t)+v_R(t) = u+k\dot{\omega}(t) \Rightarrow \begin{cases} L\,i'(t) + v_C(t) + Ri(t) = u + \dfrac{k^2}{J}i(t) - \dfrac{kD}{J}\omega \\ C\,\dot{v}_C(t) = i(t) \end{cases} \tag{2}$$

where $i(t)$ is the current of the loop, $v_C(t)$, $v_L(t)$, $v_R(t)$ are the voltage drops on the capacitor, on the inductor and on the resistor, respectively, finally $u(t)$ constitutes the control input voltage provided by a voltage source. Note that $i'(t)$ denotes the time derivative of signal $i(t)$.

Equations (1) and (2) together give the system equation:

$$\begin{cases} \dot{\omega}(t) = \dfrac{k\,i(t)}{J} - \dfrac{D\,\omega}{J} \\ \dot{v}_C(t) = \dfrac{i(t)}{C} \\ i'(t) = \dfrac{1}{L}\left(-v_C(t) - R\,i(t) + u + \dfrac{k^2}{J}i(t) - \dfrac{kD}{J}\omega\right) \end{cases} \tag{3}$$

All variables and parameters of the system equation are given in Table 1.

	State variables	**SI unit**
$i(t)$	current of the main loop	[A]
$v_C(t)$	voltage drop on the capacitor	[V]
$\omega(t)$	angular velocity of the rotor	[1/s]
	Input	
$u(t)$	voltage given by a signal generator	[V]
	System parameters	
J	moment of inertia of the rotor	[kg m^2]
k	is the motor torque constant	[N m/A]
D	is the motor viscous friction constant	[N m s]
R	electric resistance of the resistor	[Ω]
C	electric capacitance of the capacitor	[F]
L	electric inductance of the inductor	[H]

Table 1: Variables of the system equation and their SI units.

Let us denote $x_1 = \omega$, $x_2 = v_C$, $x_3 = i$ and let the output be $y = \omega$. Finally, rewrite the equations in (3) in a matrix-vector representation, which give at last the system's state space model

$$\begin{cases} \dot{x} = \begin{pmatrix} -\frac{D}{J} & 0 & \frac{k}{J} \\ 0 & 0 & \frac{1}{C} \\ -\frac{kD}{LJ} & -\frac{1}{L} & \frac{k^2}{LJ} - \frac{R}{L} \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix} u \\ y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x \end{cases} \tag{4}$$

(a) Rewrite the system in matrix-vector form with the following numbers.

$$\begin{aligned} R &= 1.5 \\ L &= 1 \\ C &= 1 \\ D &= 2 \\ J &= 1 \\ k &= 0.5 \end{aligned} \tag{5}$$

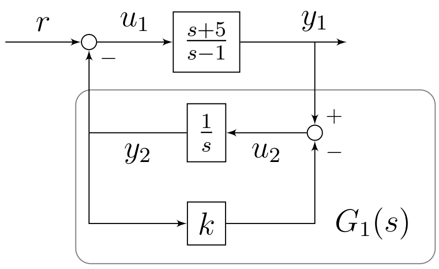
(b) Is the system exponentially stable?

(c) Explain, why the impulse response of the system is oscillating for a while.

(d) Design a static state feedback controller such that the closed loop system will not oscillate at all. For example, place the poles of the closed loop system into -1, -2 and -3.

(e) Design an observer gain L such that the error dynamics of the (Luenberger) observer $\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$ tends exponentially to zero.

Problem II. Block diagram algebra



1. Compute the resulting transfer function $G(s)$ for this block diagram. *First of all try to determine the resulting transfer function* $G_1(s)$ *of the highlighted subsystem.*

2. Choose the value of k such that the poles of the resulting transfer function be -1 and -2.