Computer Controlled Systems

Homework 2.

Submission deadline: **8th of November**, at 13:00 (approx. 3 weeks)

All solutions are expected to be calculated by hand, also all figures have to be drawn by hand. Computer programs (e.g. Matlab) can be used for self-verification, but all problems have to contain the detailed steps of solutions

Problems

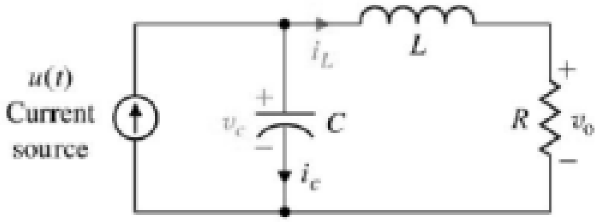

Figure 1: Parallel RLC circuit.

1. Consider the above circuit diagram. If we apply Kirchoff's laws for it, we can get the following equations

$$C\frac{\mathrm{d}v_C}{\mathrm{d}t} = -i_L + u(t)$$
$$L\frac{\mathrm{d}i_L}{\mathrm{d}t} = v_C - i_L R$$

Then, let us choose signals v_C and i_L as state variables, and the output that we would like to measure is the current of the resistor. Formally, $x_1 = v_C$, $x_2 = i_L$, $y = v_o = R\, i_R = R\, x_2$. At this point, we can set up the state-space model of the system, that is the following:

$$\dot{x}_1 = -\frac{1}{C}x_2 + \frac{1}{C}u$$
$$\dot{x}_2 = \frac{1}{L}x_1 - \frac{R}{L}x_2$$
$$y = R\, x_2$$

(a) Determine the matrices A, B, C, D, and rewrite the state-space model in matrix-vector form!

(b) Give the transfer function $H(s)$ of the system.

(c) Consider the following parameters for the circuit $R = 20\Omega$, $L = 5\mathrm{H}$, $C = 10\mathrm{mF}$, calculate the impulse-response function $h(t)$!

(d) By using the same values as in point c), determine the eigenvalues of matrix A. Is the system stable?

(e) The energy stored in the linear passive dynamic elements are the following:

the energy stored in the capacitor: $E_C = \frac{1}{2}C\, v_c^2$

the energy stored in the inductance: $E_L = \frac{1}{2}L\, i_L^2$

Check that the total energy function $V = E_C + E_L$ is an appropriate Lyapunov function (satisfying the prescribed Lyapunov conditions: $V(x) > 0$ and $\dot{V}(x) < 0$ for all $x \neq 0$.

2. We consider a physical system that contains four cascaded water tanks. Figure 2 helps to imagine it. We consider an additional water flow to the second water tank as an input, furthermore, we measure the level of the third water tank. The dynamics of

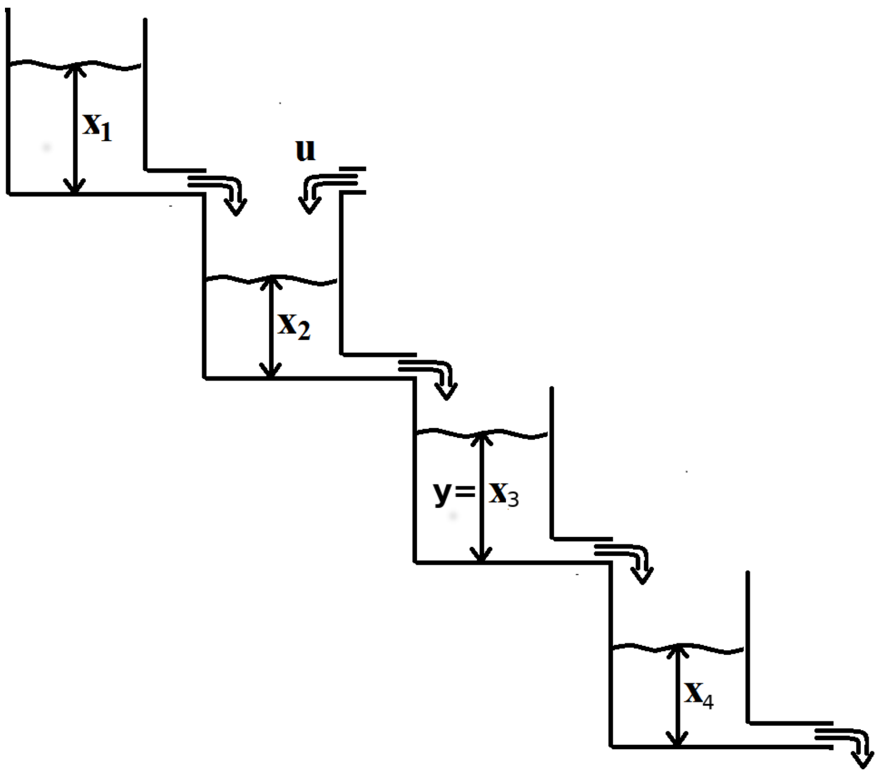

Figure 2: Cascaded water tanks.

the water levels in each tank can be modeled by the following linear time-invariant dynamic equation:

$$\begin{cases} \dot{x_1} = -k_1 x_1 \\ \dot{x_2} = k_1 x_1 - k_2 x_2 + u \\ \dot{x_3} = k_2 x_2 - k_3 x_3 \\ \dot{x_4} = k_3 x_3 - k_4 x_4 \\ y = x_3 \end{cases} \tag{1}$$

The units of the parameters are: $x_n[m]$ water level in n-th tank; $k_n[\frac{1}{h}]$ water outflow coefficient of the n-th tank; $u[\frac{m^3}{h}]$ water inflow rate to the second tank.

(a) Please rewrite the state-space model in matrix-vector form!

(b) Determine the controllability and observability matrices of system!

(c) Compute the controllable subspace and the unobservable subspace of the system!

(d) Discuss the physical interpretation of the obtained subspaces!

[only TP$_1$] Give a minimal realization of the system without computing the transfer function (only considering the unobservable/uncontrollable state variables).