Computer Controlled Systems

Homework 1.

Submission deadline: **25th of October**, at 13:00 (approx. 3 weeks)

All solutions are expected to be calculated by hand, also all figures have to be drawn by hand. Computer programs (e.g. Matlab) can be used for self-verification, but all problems have to contain the detailed steps of solutions

Problems

1. Given a linear mapping $\mathcal{A}$: $\mathbb{R}^3 \to \mathbb{R}^3$, $\mathcal{A}(v) = Av$, where matrix A is given as follows:
$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$
   (a) Apply the transformation for vector $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \in \mathbb{R}^3$. What does the transformation do with its coordinates?
   (b) Calculate the eigenvalues and the corresponding eigenvectors of matrix A!
   (c) Determine the diagonal matrix D of mapping $\mathcal{A}$ and give an appropriate coordinate transformation matrix S such that $D = S^{-1} \cdot A \cdot S$.
   (d) Compute the exponential matrix e^D!
   (e) Compute the exponential matrix e^A!

2. It is given the following matrix $B = \begin{pmatrix} 1 & -1 & 0 & 22 \\ 0 & 1 & -2 & 5 \\ -3 & 2 & 5 & -65 \\ -2 & 6 & 4 & 0 \end{pmatrix}$. Determine the image space and kernel space of matrix B (in the canonic basis)!

3. It is given the following differential equation $\ddot{y} + 4\dot{y} + 3y = u(t)$ with initial conditions $\dot{y}(0) = -1, y(0) = 1$. Determine $y(t)$ using Laplace transformation, if the input is
   (a) $u(t) = 4e^{-2t}$

   Compulsory only for TP students, ***but extra points for others***. It is advised to help your computations with Matlab and/or other symbolic math software tool.

   (b) $u(t) = \sin 5t$
   (c) $u(t) = e^{-2t} \cos 5t$.

4. Let us consider an LTI system with the following state space model:
$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}, \text{ where } A = \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \; C = \begin{pmatrix} 1 & -1 \end{pmatrix}. \tag{1}$$
   (a) Determine the transfer function $H(s) = \frac{Y(s)}{U(s)}$ of the system using Laplace transformation.
   (b) Give the impulse-response function $h(t)$ of the system!