Computer controlled systems

Matrix algebra

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Proposition 1. If $A \in \mathbb{R}^{n\times n}$ and $D \in \mathbb{R}^{m\times m}$ are nonsingular, than the determinant of the following block-triangle matrix can be computed as follows:

$$\begin{vmatrix} A & 0 \\ C & D \end{vmatrix} = \det(A) \cdot \det(D) \tag{1}$$

Proof.

$$\begin{pmatrix} I & 0 \\ -CA^{-1} & I \end{pmatrix} \begin{pmatrix} A & 0 \\ C & D \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix} \tag{2}$$

□

Proposition 2. If $A \in \mathbb{R}^{n\times n}$ and $D \in \mathbb{R}^{m\times m}$ are nonsingular, than the determinant of the following block matrix can be computed as follows:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \Rightarrow \det(M) = \det\big(A - BD^{-1}C\big)\det(D). \tag{3}$$

If $m = n$ and if C and D commute then $\det(M) = \det(AD - BC)$.

Proof.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} I & 0 \\ -D^{-1}C & I \end{pmatrix} = \begin{pmatrix} A - BD^{-1}C & B \\ 0 & D \end{pmatrix} \tag{4}$$

□