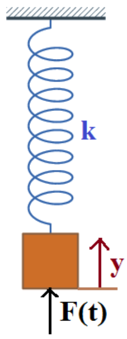
Computer Controlled Systems
Homework 4
Submission deadline: December 14. 2017. 10:00/12:00 (end of the seminar)

All solutions are expected to be calculated by hand, also all figures have to be drawn by hand. Computer programs can be used for self-verification, but all problems have to contain the detailed steps of solutions!

Pole placement controller design for a spring-mass system

Consider a simple undamped spring-mass system with an external force $F(t)$. The time-dependent position of the mass is $y(t)$, where the zero position $y = 0$ belongs to the relaxed state of the spring.



The differential equation that describes the dynamics of the spring-mass system is

$$\ddot{y}(t) + \frac{k}{m}y(t) = \frac{1}{m}F(t) \qquad \dot{y}(0) = 0 \quad y(0) = 0$$

By setting the input variable as $u(t) = F(t)$, with the values of system constants $k = 0.2Nm^{-1}$ and $m = 0.1kg$ the input-output model of the spring-mass system is as follows:

$$\ddot{y}(t) + 2y(t) = 10u(t) \tag{1}$$

By choosing the state variables as the position of the mass ($x_2 = y$) and the velocity of the mass ($x_1 = \dot{x}_2$) a second order LTI state space realization can be given:

$$\begin{aligned} \dot{x}_1(t) &= -2x_2(t) + 10u(t) \\ \dot{x}_2(t) &= x_1(t) \\ y(t) &= x_2(t) \end{aligned}$$

where $x_1(0) = 0$ and $x_2(0) = 0$.

This system responds to perturbations with undamped oscillations. In order to remove the oscillatory behaviour, and to asymptotically stabilitize the system, a simple pole placement controller can be applied.

1. Write this state-space model in matrix-vector form!
2. Check the asymptotic stability of the system! What refers to the oscillatory behaviour?
3. Is it possible to design a pole placement controller for this system?
4. Construct a pole placement controller that stabilizes the system with the prescribed poles $\lambda_1 = -3$ and $\lambda_2 = -4$!
5. Check your result by the eigenvalues of the state matrix of the closed loop system!

State observer design for a spring-mass system

The pole placement controller designed uses both state variables, although the time function of the velocity of the mass ($x_1(t)$) is unknown. To solve this problem, a state observer can be designed.

1. Is it possible to design a state observer for this system?
2. Construct an observer to estimate the state variables, with poles $\lambda_1 = -5$ and $\lambda_2 = -10$!