# Computer controlled systems

Exercises for practice

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1. Design a state observer gain $L$ for the following state space model:

$$A = \begin{pmatrix} -1 & -2 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -3 & -1 \end{pmatrix}$$

$$\text{answer: } L^T = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{O}_n^{-T} = \left( \begin{pmatrix} 4 & 3 \end{pmatrix} - \begin{pmatrix} 0 & -1 \end{pmatrix} \right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -4 & 4 \end{pmatrix}$$

2. Design a full state feedback gain $K$ for the following state space model:

$$A = \begin{pmatrix} -1 & 0 \\ -2 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -3 & -1 \end{pmatrix}$$

$$\text{answer: } K = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{C}_n^{-1} = \left( \begin{pmatrix} 4 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 1 \end{pmatrix} \right) \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -\frac{1}{2} \\ 1 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2 & 0 \end{pmatrix}$$

3. Design a full state feedback gain $K$ for the following state space model:

$$A = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -4 & -3 \end{pmatrix}$$

$$\text{answer: } K = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{C}_n^{-1} = \left( \begin{pmatrix} 7 & 12 \end{pmatrix} - \begin{pmatrix} 2 & 1 \end{pmatrix} \right) \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 11 & -17 \end{pmatrix}$$

4. Design a full state feedback gain $K$ for the following state space model:

$$A = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -4 & -1 \end{pmatrix}$$

$$\text{answer: } K = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{C}_n^{-1} = \left( \begin{pmatrix} 5 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 1 \end{pmatrix} \right) \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & -1 \end{pmatrix} = \begin{pmatrix} 0 & 3 \end{pmatrix}$$

5. Design a full state feedback gain $K$ for the following state space model:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -2 \\ -1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -4 & -3 \end{pmatrix}$$

$$\text{answer: } K = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{C}_n^{-1} = \left( \begin{pmatrix} 7 & 12 \end{pmatrix} - \begin{pmatrix} 2 & -1 \end{pmatrix} \right) \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 1 \end{pmatrix}$$

6. Design a full state feedback gain $K$ for the following state space model:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -1 & -4 \end{pmatrix}$$

$$\text{answer: } K = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{C}_n^{-1} = \left( \begin{pmatrix} 5 & 4 \end{pmatrix} - \begin{pmatrix} -1 & -1 \end{pmatrix} \right) \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \end{pmatrix}$$

7. Design a state observer gain $L$ for the following state space model:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -1 & -3 \end{pmatrix}$$

$$\text{answer: } L^T = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{O}_n^{-T} = \left( \begin{pmatrix} 4 & 3 \end{pmatrix} - \begin{pmatrix} -1 & -1 \end{pmatrix} \right) \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \end{pmatrix}$$

8. Design a full state feedback gain $K$ for the following state space model:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -2 \\ -1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -2 & -2 \end{pmatrix}$$

answer: $K = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{C}_n^{-1} = \left( \begin{pmatrix} 4 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 \end{pmatrix} \right) \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \end{pmatrix}$

9. Design a full state feedback gain $K$ for the following state space model:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -2 & -4 \end{pmatrix}$$

answer: $K = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{C}_n^{-1} = \left( \begin{pmatrix} 6 & 8 \end{pmatrix} - \begin{pmatrix} -2 & 1 \end{pmatrix} \right) \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 15 & -22 \end{pmatrix}$

10. Design a full state feedback gain $K$ for the following state space model:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \end{pmatrix}, \quad \text{desired poles: } p = \begin{pmatrix} -4 & -4 \end{pmatrix}$$

answer: $K = (\boldsymbol{\alpha} - \boldsymbol{a})T_l^{-1}\mathcal{C}_n^{-1} = \left( \begin{pmatrix} 8 & 16 \end{pmatrix} - \begin{pmatrix} -1 & -1 \end{pmatrix} \right) \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -25 & 8 \end{pmatrix}$

11. Determine the overall transfer function of the following block diagram! (5p)