Computer Controlled Systems

supplementary midterm test

2018. 12. 20.

theoretical questions (25 points)

(The answers can be given in Hungarian)

1. When do we call a continuous time LTI state space model (A, B, C) controllable (i.e. what is the definition of controllability)? (2p)
   What is the necessary and sufficient condition for controllability? (2p)
   What is the controllable subspace of (A, B, C), and how can we compute it? (2p)

2. Consider the following continuous time LTI model

$$\dot{x} = Ax + Bu \tag{1}$$

   where

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

   Let $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

   (a) Show that $A^T P + PA$ is negative definite. (3p)

   (b) Give a Lyapunov function for the system model in Eq. (1), and check that its derivative is negative whenever $x \neq 0$. (2+2p)

3. Give the problem statement of pole placement control (what data are given, what is the design goal, and what is to be computed)? (4p)
   What equation should be solved for the controller design and what kind of feedback is obtained as a result? (2p)

4. Describe briefly zero order hold sampling. (2p)
   Give the general form of discrete time LTI state space models. Give the dimensions of variables and matrices in the model. How do the matrices of the discrete time model depend on the matrices of the continuous time state space model and the sampling time? (4p)