# Computer Controlled Systems

supplementary midterm test – 2017. 05. 25.

(The answers can be given in Hungarian)

*Theoretical questions* (25 points)

1. How can we compute the response $y(t)$ of a continuous time linear time invariant (CT-LTI) system, if the input is the Dirac-$\delta$ function ($u(t) = \delta(t)$), and we know the state-space representation $(A, B, C, D)$ of the system? (We assume that the initial condition $x(0)$ is zero.) (5p)

2. Describe the conditions, that a continuous-time Lyapunov function has to satisfy. (You do not need to state the Lyapunov theorem.) (5p)

3. When do we call a CT-LTI state space model $(A, B, C)$ controllable? What is the necessary and sufficient condition for controllability? (5p)

4. Briefly describe PID control (controller structure, transfer function, parameters, block scheme of the whole PID control loop). (5p)

5. Write down the state space model of a discrete-time linear time invariant (DT-LTI) system. Give the dimensions of the vectors and matrices. How do the matrices of the DT-LTI model depend on the matrices of the CT-LTI system? (5p)

*Computational exercises* (25 points)

1. Consider the state space model $\left\{ \begin{smallmatrix} \dot{x} = Ax + Bu \\ y = Cx \end{smallmatrix} \right.$, where $A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 1 \end{pmatrix}$.

   (a) Give the transfer function $H(s)$ for the system. (3p)

   (b) Is the system asymptotically stable? Is the system BIBO stable? Is the system observable? Is the system and controllable? Is the state space model minimal? Justify your answers. (5p: 1p for each question)

2. The following continuous-time state space model is given:

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 \end{pmatrix} \tag{1}$$

   Determine the model matrices $\Phi$ and $\Gamma$ of the discretized state-space model

$$x(k+1) = \Phi x(k) + \Gamma u(k), \quad y(k) = Cx(k),$$

   if the sampling period is $h = \ln(3)$. (5p)

3. Consider the following transfer function:

$$H(s) = \frac{s^2 + 1}{s^3 + 3s + 3}$$

   (a) Assume that $u(t) = 6$, $\forall t \geq 0$. To which value will the output ($y$) converge when $t \to \infty$? (In other words: determine $\lim_{t\to\infty} y(t)$.) (3p)

   (b) Give the controller form state space realization of the system. (4p)

4. Determine the overall transfer function $G_e(s)$ (from $r$ to $y$)! (5p)