Name: ....

Neptun code: ....

1:	2:
3:	Σ:

Computer Controlled Systems

1st midterm test

2019. 10. 22.

computational problems (25 points)

(The answers can be given in Hungarian)

In case of yes-no questions, please, always justify your answer!

The test-sheet should be submitted with the computational part (white papers)!

1. We consider a SISO LTI system given by the following state-space model ($\Sigma = 10pt$)

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t), \end{cases} \tag{i}$$

where $A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ -1 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$, $D = 0$

   (a) Give a possible state value, which can not be reached using any input functions. (3p)

   (b) Compute the unobservable subspace of system (i). (3p)

   (c) Determine the transfer function for this system. (2p)

   (d) Notice that there is a set containment relation between the unobservable subspace and the controllable subspace. Then, try to explain the obtain specific value for the transfer function. How the output is affected by the input? (2p)

2. It is given the following second order system of linear differential equations: ($\Sigma = 10pt$)

$$\begin{cases} \ddot{z}(t) + 3\dot{z}(t) + 2z(t) = u(t), \\ \dot{y}(t) + 3y(t) = z(t) \end{cases} \tag{ii}$$

where $u(t)$ is the input signal, $y(t)$ is the output signal and $z(t)$ is an internal signal.

   (a) Assume that $z(0) = 0$, $y(0) = 0$ but $\boldsymbol{\dot{z}(0) = a \neq 0}$. Perform a Laplace transformation on both equations in (ii) and express $Y(s) = \mathcal{L}\{y(t), s\}$ as a function of $U(s) = \mathcal{L}\{u(t), s\}$ and of the initial value $\dot{z}(0) = a$. (3p)

   (b) Determine $y(t)$ if the input is $u(t) = e^{-4t}$ and the initial conditions is $z(0) = 0$, $y(0) = 0$, $\dot{z}(0) = a = 1$. (3p)

   (c) Determine the transfer function $H(s) = \frac{Y(s)}{U(s)}$ for this system ($a = 0$). (2p)

   (d) Give a state space representation for this system. (2p)

3. We consider the mass-spring-damper system ($\Sigma = 5pt$)

$$m\ddot{y} + D\dot{y} + ky^3 = 0, \tag{iii}$$

with a nonlinear spring characteristics $F_k(y) = -ky^3$. Let $m = 1$, $D = 1$, $k = 1$. We consider a Lyapunov function of the form:

$$V(x) = \frac{(x_1 + x_2)^2}{2} + ax_1^4, \text{ where } x_1 = y,\ x_2 = \dot{y}. \tag{iv}$$

Coefficient a is a free variable and is meant to be selected such that $V(x)$ is a Lyapunov function for (iii). Determine the value of a such that $V(x)$ satisfies the Lyapunov conditions (i.e. with $V(x)$ we can prove global stability for system (iii)).