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Name: .....

Neptun code: .....

**Computer Controlled Systems**

1st midterm test

2019. 10. 22.

*computational problems* (25 points)

(The answers can be given in Hungarian)

← Required layout for each submitted paper!

**In case of yes-no questions, please, always justify your answer!**

1. We consider a SISO LTI system given by the following state-space model ($\Sigma = 10pt$)

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t), \end{cases} \tag{i}$$

where $A = \begin{pmatrix} 1 & 0 & 0 \\ -3 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}$ , $B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ , $C = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$ , $D = 0$

   (a) Compute the unobservable subspace of system (i). (4p)

   (b) Give two different state values, which can not be distinguished from each other by measuring only $y(t) = Cx(t)$. (1p)

   (c) Give a minimal state space realization for this system. (5p)

2. It is given the following state space model: ($\Sigma = 10pt$)

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -8x_1(t) - 6x_2(t) + 3u(t) \\ \text{with } y(t) = x_1(t) \end{cases} \tag{ii}$$

where $u(t)$ is the input signal, $y(t)$ is the output signal.

   (a) Determine the model matrices $(A, B, C)$ of system (ii). (1p)

   (b) Determine the transfer function $H(s)$ for this system. (3p)

   (c) Using Laplace transformation determine the value of $Y(s) = \mathcal{L}\{y(t), s\}$ if the input is $u(t) = -2e^{-3t}$ and the initial state values are $x_1(0) = 0$ and $x_2(0) = 5$. (3p)

   (d) Compute the output $y(t)$ if the input and the initial state values are the same as in the previous point (c). (3p)

3. It is given the following nonlinear state-space model ($\Sigma = 5pt$)

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3^2 - x_1, \\ \dot{x}_2 = -2x_3(x_3^2 - x_1), \end{cases} \tag{iii}$$

and a Lyapunov function candidate:

$$V(x) = \frac{x_2^2}{2} + \frac{(x_3^2 - x_1)^2}{2}. \tag{iv}$$

Check, whether $V(x)$ satisfies the Lyapunov conditions (i.e. with $V(x)$ we can prove global stability for system (iii)).