Computer Controlled Systems

1st midterm test

2019. 10. 22.

theoretical problems (25 points)

(The answers can be given in Hungarian)

In case of yes-no questions, please, always justify your answer!

1. (a) Let $f, g : \mathbb{R}_0^+ \to \mathbb{R}$. Define the causal convolution of f and g. (2p)

   (b) Give the value ($e^{A\cdot 1}$) of the exponential matrix function at $t = 1$ of the following matrix:

$$A = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

   where $\lambda_1, \lambda_2 \in \mathbb{R}$. (3p)

2. Let the transfer function of an LTI system be the following:

$$H(s) = \frac{s-1}{(s^2-1)(s+2)}.$$

   Decide whether the third order observer realization for $H(s)$ is controllable or not? (5p)

3. When do we call a state space model (A, B, C) controllable? What is the necessary and sufficient condition for controllability? What is the controllable subspace of (A, B, C), and how can we compute it? (5p)

4. (a) Define the notion of Markov-parameters corresponding to a state-space model (A, B, C). Do the Markov parameters change if we apply a state transformation $\bar{x} = Tx$ to the system? (2p)

   (b) Define the notion of a positive definite matrix. How can we computationally check (e.g. with Matlab) whether a matrix is positive definite? (3p)

5. When do we call an LTI system BIBO stable? What is the necessary and sufficient condition for BIBO stability? (5p)

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2019. 10. 25.

theoretical problems (25 points)

(The answers can be given in Hungarian)

In case of yes-no questions, please, always justify your answer!

1. (a) Let $f(t) = e^{-t} + 1$. Determine the value of the following integral:

$$\int_0^t f(\tau)\delta(t-\tau)dt,$$

   where δ is the Dirac-delta function. (2p)

   (b) Let $h(t) = e^{-t} + 2e^{3t}$ denote the impulse response function of a certain LTI system. Give the transfer function for this system. (3p)

2. (a) Define the notion of an $n \times n$ stability matrix. (2p)

   (b) The eigenvalues of a 2×2 matrix A are the following: $\lambda_1 = 1$, $\lambda_2 = 2$. Determine the eigenvalues of the following matrix polynomial $A^{10} - 12A$? (3p)

3. Consider the autonomous nonlinear system: $\dot{x} = f(x)$, where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}^n$.

   (a) When do we call $x^* \in \mathbb{R}^n$ an equilibrium point of the system? (2p)

   (b) When do we say that x^* is a *stable* equilibrium point? (3p)

4. When do we call a state space model (A, B, C) observable? What is the necessary and sufficient condition for observability? What is the unobservable subspace ($\mathcal{X}_{\bar{o}}$) of (A, B, C)? Determine the value of $y_1 = Cx_1$, if $x_1 \in \mathcal{X}_{\bar{o}}$. (5p)

5. Give a possible state space realization for the following transfer function model: (1p)

$$H(s) = \frac{s^2 - 1}{(s^2 + 3s + 2)(s - 1)}$$

   (Remark. no calculations are needed to answer the following questions:)

   (a) Is the computed realization observable, controllable, minimal, asymptotically stable, BIBO stable? Why? (3p)

   (b) Which of these properties (observability, controllability, minimality, asymptotic stability, BIBO stability) are independent of the state space realization. (1p)