Computer Controlled Systems

2nd midterm test

2017.12.07.

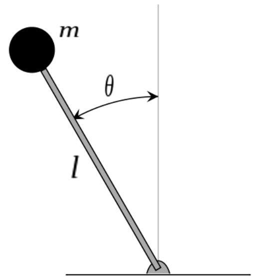
computational exercises (25 points)

The answers can be given in Hungarian.

Problems marked by ** **are a bit tricky or they require a deeper theoretical comprehension, but they are also compulsory for everybody.**

Let the lower end of an inverted pendulum be fixed to a joint as presented in the figure, furthermore, we consider an external angular momentum $\vec{L}(t)$ applied to the the pendulum ($\vec{L}(t)$ is parallel to the axis of rotation of the joint). For small variations of $\theta(t)$, we may assume that $\sin\big(\theta(t)\big) \simeq \theta(t)$, thus the dynamic equation of the pendulum is: $ml^2\,\ddot{\theta}(t) - mgl\theta(t) = \dot{L}(t)$, where

- $m = 0.25$ kg is the mass of the pendulum,
- $l = 2$ m is length of the pendulum,
- $g = 10$ m/s^2 is the gravitational acceleration,
- $\theta(t)$ is the angle of the pendulum in radians at time t,
- $L(t)$ is the signed length of the angular momentum vector at time t.



Problems. **(They can be solved independently of each other!)**

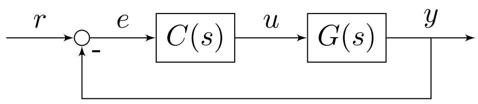
(2p) 1. Considering $u(t) = L(t)$ as the input and $y(t) = \theta(t)$ as the output of the system, show that the transfer function from u to y is:

$$G(s) = \frac{s}{s^2 - 5}. \qquad \text{(Eq1)}$$

(2p) 2.** Show that the state space representation of the system is

$$\begin{cases} \dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 5 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t), \\ y(t) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(t), \end{cases} \quad \text{where } x(t) = \begin{pmatrix} \int_0^t \theta(\tau)\mathrm{d}\tau \\ \theta(t) \end{pmatrix}. \qquad \text{(Eq2)}$$

(3p) 3. In order to control system $G(s)$, we consider the following control loop:



Give the resulting transfer function $G_e(s)$ of the overall closed loop if the controller's transfer function is given in the parametric form $C(s) = \dfrac{k_1}{s} + k_2$.

(3p) 4. Give a possible value for both $k_1 \neq 0$ and $k_2 \neq 0$, such that the closed loop system is BIBO stable.

(3p) 5.** Give a particular transfer function for $C(s)$ which ensures that the output $y(t)$ of the overall system follows the reference DC signal $r(t)$. *Hint.* Remember, how would you compute the DC-gain (i.e. the limit of the step response) of the system.

(5p) 6. Design a pole-placement controller (with gain K) for the state space model (Eq2), such that the desired characteristic polynomial of the controlled system is $\alpha(s) = s^2 + s + 1$.

(5p) 7. Design a state observer with gain L for system (Eq2) with the following prescribed poles: -1 and -2.

(5p) 8. The figure on the right illustrates a nested control loop for system $G(s)$ of (Eq1). Compute the resulting transfer function and determine whether the overall system is BIBO stable or not.