# Computer Controlled Systems

1st midterm test
2018. 10. 25.
*computational exercises* (25 points)
(The answers can be given in Hungarian)

1. A linear state space model is given with the following matrices ($D = 0$): **(7p)**

$$A = \begin{pmatrix} -2 & 0 \\ 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 \end{pmatrix}$$

   (a) Can we determine the value of the state vector from a finite measurement of the inputs and outputs? (2p)

   (b) Determine the output of the system if the input is the Dirac-delta function. (2p)

   (c) Give a particular value of the state vector $x$ that cannot be reached by any input from the origin. (3p)

2. We consider a continuous-time LTI system given by its transfer function **(4p)**

$$H(s) = \frac{s^2 - 1}{(s^2 + 5s + 6)(2s - 2)}$$

   (a) Give a jointly controllable and observable state-space realization for this system. (2p)

   (b) Determine whether the system is BIBO stable or not. (2p)

3. Compute the step response of the following system given by its transfer function: **(7p)**

$$H(s) = \frac{s}{s^2 + 3s + 2}$$

   In other words, compute $y(t)$ if the input is the unit step function $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$.

   The Laplace transform of $u(t)$ is $\mathcal{L}\{u(t), s\} = 1/s$.

4. The following nonlinear state-space model is given **(7p)**

$$\Sigma : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -x_1 - x_2 - \sin(2x_1), \end{cases}$$

   and a Lyapunov function candidate:

$$V(x) = \frac{1}{2}(x_1^2 + x_2^2) + a \sin^2(x_1).$$

   Give a possible value for the parameter $a$, such that $V(x)$ satisfies the Lyapunov conditions (i.e. with $V(x)$ we can prove global stability for system $\Sigma$).