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Computer Controlled Systems

1st midterm test

2017. 10. 19.

computational problems (25 points)

(The answers can be given in Hungarian)

In case of yes-no questions, please, always justify your answer!

The test-sheet should be submitted as well!

1. We consider a SISO LTI system given by the following state-space model ($\Sigma = 10pt$)

$$A = \begin{pmatrix} 3 & -4 \\ 2 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & 3 \end{pmatrix}, \quad D = 0 \tag{i}$$

   (a) Compute the transfer function $H(s)$ for the system (A, B, C). (2p)

   (b) Is the system globally asymptotically stable? (2p)

   (c) Is the system BIBO stable? (2p)

   (d) Is the state-space model (A, B, C) minimal? (2p)

   (e) Compute the controllable subspace of (A, B, C). (2p)

2. It is given the following second order linear differential equation: ($\Sigma = 10pt$)

$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = u(t), \tag{ii}$$

   where $u(t)$ is the input signal, $y(t)$ is the output signal.

   (a) Determine the transfer function $H(s)$ for this system, i.e. $Y(s) = H(s)U(s)$. (2pt) (All initial conditions are zero: $y(0) = 0$, $\dot{y}(0) = 0$.)

   (b) Solve the differential equation (ii), if the input is $u(t) = -2e^{-3t}$ ($y(t) = ?$). (4pt) Initial conditions: $y(0) = 0$, $\dot{y}(0) = 2$.

   (c) Give a minimal state-space representation for system (ii). (2pt)

   (d) Compute the DC-gain of system (ii). (2pt)
   Hint. DC-gain $= \lim_{t\to\infty} y(t)$ if the input $u(t) = 1(t)$ is the unit step function. Furthermore, $\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$, where $Y(s)$ is the Laplace transform of $y(t)$.

3. It is given the following nonlinear state-space model ($\Sigma = 5pt$)

$$\begin{aligned} \dot{x}_1 &= -x_1 - 2x_2, \\ \dot{x}_2 &= 4x_1^3 - x_2, \end{aligned} \tag{iii}$$

   and a Lyapunov function candidate:

$$V(x) = ax_1^4 + x_2^2. \tag{iv}$$

   Give a possible value for the parameter a, such that $V(x)$ satisfies the Lyapunov conditions (i.e. with $V(x)$ we can prove global stability for system (iii)).