Computer Controlled Systems

1st midterm test
2017. 10. 19.
theoretical questions (25 points)
(The answers can be given in Hungarian)

1. Define the exponential function e^{At} of a square matrix A using power series. Give an explicit formula to compute e^{At} analytically (not only approximately). (5p)

   Solution: $e^{At} = \sum_{i=0}^{\infty} \frac{A^i t^i}{i!}$
   $e^{At} = \mathcal{L}^{-1}\left\{(sI - A)^{-1}\right\}(t)$

2. Define the impulse response function h of a SISO linear time invariant (LTI) system. How can we compute h from the matrices (A, B, C) of a state space model? (5p)

   Solution: h is the system response to the Dirac-δ input. Or: $y = h * u$ ($Y(s) = H(s)U(s)$) for any input u.
   $h(t) = Ce^{At}B$, or: $H(s) = C(sI - A)^{-1}B$

3. When do we call an LTI system BIBO stable? What is the necessary and sufficient condition for BIBO stability? (5p)

   Solution: For any bounded input, a BIBO stable system produces a bounded output.
   $\int_0^{\infty} |h(t)| dt \leq M < \infty$ (impulse response function is absolute integrable)

4. Let $A \in \mathbb{R}^{n \times n}$ be a positive definite symmetric matrix. Can A have complex conjugate eigenvalues? Does there exist an invertible transformation matrix $T \in \mathbb{R}^{n \times n}$, such that $\bar{A} = TAT^{-1}$ is a stability matrix? Justify your answers! (5p)

   Solution: No, because a symmetric matrix always has real eigenvalues.
   No, because a positive definite symmetric matrix has positive eigenvalues and these are not changed during transformation of bases.

5. Define the notion of observability of a state space model (A, B, C). Define the unobservable subspace of (A, B, C). Determine the unobservabe subspace of the following second order LTI system:

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 \\ \dot{x}_2 &= a_{22}x_2 + b_2 u \\ y &= c_2 x_2 \qquad\qquad c_2 \neq 0 \end{aligned} \qquad (5p)$$

   Solution: Observable system: we can compute the state x knowing the system model (A, B, C) and a finite measurement of inputs and outputs.

   Unobservable subspace: the set of initial conditions for which the system gives the same output for a given input. (Computation: $\ker(\mathcal{O}_n)$)

   Unobservable subspace of the example: $\lambda \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\lambda \in \mathbb{R}$