Computer Controlled Systems

2nd midterm test

2017. 05. 12.

computational exercises (25 points)

(The answers can be given in Hungarian)

(7p) 1. Consider the following continuous time state space model:

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

- (3p) a) Design a stabilizing state feedback, which moves the poles of the system into $[-1, -2]$.
- (2p) b) Design a state observer (L) with the following prescribed poles $[-2, -2]$.
- (2p) c) Give the state space model of the observer.

(5p) 2. The following continuous-time state space model is given:

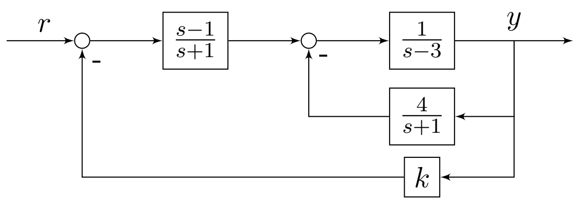
$$A = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \end{pmatrix} \tag{1}$$

Determine the model matrices Φ and Γ of the discretized state-space model

$$x(k+1) = \Phi x(k) + \Gamma u(k), \quad y(k) = Cx(k),$$

if the sampling period is $h = \ln(2)$.

(5p) 3. Consider the following feedback system



- (3p) a) Determine the overall transfer function $G_e(s)$ (from r to y)!
- (2p) b) For which values of $k > 0$ is the system BIBO stable?

(5p) 4. It is a well-known fact that the rabbits' reproduction rate is exponential in the absence of predators (assuming unlimited food and space). Design a PI controller such that the number of rabbits converges to a constant reference value.

The dynamics of the rabbit population is given by the model $\boxed{\dot{x} = 2x + u}$ (1)
x: number of rabbits (transformed into a continuous variable)
u: manipulable input (hunting/feeding rate)

- (1p) a) Give the transfer function $H(s)$ for the system (1)
- (4p) b) Design a PI controller $H_{\mathrm{PI}}(s) = K_p + \frac{K_i}{s}$, which stabilizes the system and ensures reference tracking. (Determine K_p and K_i.)

(3p) 5. The nonlinear state-space equation of a pendulum is the following:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_2 - \sin x_1, \end{aligned} \tag{2}$$

where x_1 is the angle and x_2 is the angular velocity of the rod. For this system, $x^* = (x_1, x_2) = (0, 0)$ is a locally assymptotically stable equilibrium point. In order to prove that x^* is asymptotically stable on $(-\pi, \pi) \times \mathbb{R}$, we consider the Lyapunov function candidate $V(x) = ax_2^2 + (1 - \cos x_1)$. Determine the value of $a > 0$, such that function $V(x)$ satisfy the Lyapunov conditions.