Computer Controlled Systems

1st midterm test

2017. 03. 22.

theoretical questions (25 points)

(The answers can be given in Hungarian)

1. (a) Define the notion of an $n \times n$ stability matrix. (2p)

   (b) Define the notion of a positive definite matrix. How can we computationally check (e.g. with Matlab) whether a matrix is positive definite? (3p)

2. (a) Define the Dirac-δ function. (2p)

   (b) Define the notion of Markov-parameters corresponding to a state-space model (A, B, C). Do the Markov parameters change if we apply a state transformation $\bar{x} = Tx$ to the system? (3p)

3. When do we call a state space model (A, B, C) controllable? What is the necessary and sufficient condition for controllability? What is the controllable subspace of (A, B, C), and how can we compute it? (5p)

4. Consider the state space model

$$\dot{x} = Ax + Bu$$
$$y = Cx,$$

   where

$$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

   Is the system asymptotically stable? Is the system BIBO stable? Is the system observable? Is the state space model minimal? Justify your answers. (5p)

5. (a) Let $f, g : \mathbb{R}_0^+ \to \mathbb{R}$. Define the causal convolution of f and g. (2p)

   (b) Compute the value of the exponential matrix function at time $t = 2$ (i.e. $e^{A\cdot 2}$) for the following matrix:

$$A = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

   where $\lambda \in \mathbb{R}$. Can you choose the value of λ such that $\det(e^{A\cdot t}) < 0$ for $t \geq 0$? (3p)