Computer Controlled Systems

1st midterm test

2017. 03. 24.

computational problems (25 points)

(The answers can be given in Hungarian)

We consider a SISO LTI system given by the following state-space model

$$A = \begin{pmatrix} -1 & 4 & 0 \\ 0 & -3 & 0 \\ 1 & 4 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 2 \end{pmatrix}, \quad D = 0 \tag{1}$$

1. Give the matrices of the transformed model $(\bar{A}, \bar{B}, \bar{C})$ if we apply the $\bar{x} = Tx$ state transformation, where

$$T = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix}, \quad \begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u \\ \bar{y} &= \bar{C}\bar{y} \end{aligned}$$

(4p)

2. Give the eigenvalues and the corresponding eigenvectors of matrix A! (2p)

3. Is the system globally asymptotically stable? Why? (2p)

4. Is the state-space model (A, B, C) minimal? Why? (2p)

5. Compute the controllable subspace of (A, B, C). (3p)

6. Determine the transfer function $H(s)$ of the system. (2p)

7. Compute the impulse response $h(t)$ of the system. (2p)

8. Determine a jointly controllable and observable state space representation for this system. (2p)

9. Compute the output $y(t)$ of the system $(\bar{A}, \bar{B}, \bar{C})$ if the initial condition is $\bar{x}(0) = \left(\begin{smallmatrix} 1 \\ 2 \\ 3 \end{smallmatrix}\right)$ and the input is the Dirac-delta function: $u(t) = \delta(t)$. (2p)

10. Compute the DC gain of the system.
    Remember, DC gain: $\lim\limits_{t \to \infty} y(t)$ if the input is the step function $u(t) = 1(t)$. (2p)

11. Does there exist an input $u(t)$ for $t \in [0, t_1]$, which can drive the state of the system (A, B, C) from $x(0) = \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix}\right)$ to the following final states:

    (a) $x(t_1) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, (b) $x(t_1) = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$?

(2p)