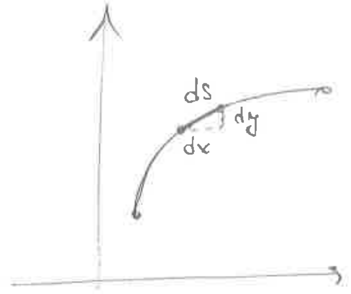
$$\int_\Gamma f(x,y)\,ds = \int_0^{\frac{\pi}{2}} \underbrace{f(x(t),y(t))}_{\gamma(t)} \underbrace{\underbrace{\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}}_{\|\dot\gamma(t)\|}\,dt}_{ds}$$

Th Garrity
91 old



$$ds = \sqrt{dx^2+dy^2}\cdot\frac{dt}{dt}$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$f(x,y) = x^2+y^2$$

$$\gamma(t) = \begin{pmatrix} a\cos t \\ b\sin t \end{pmatrix}$$

$$\int_0^{\frac{\pi}{2}} (a^2\cos^2 t + b^2\sin^2 t)\sqrt{a^2\sin^2 t + b^2\cos^2 t}\,dt$$

Afl. $a=b \Rightarrow \int_0^{\frac{\pi}{2}} a^2 a\,dt = a^3\frac{\pi}{2}$

$$F(x,y) = \begin{pmatrix} x^2 \\ y^2 \end{pmatrix} \qquad \gamma = \begin{pmatrix} a\cos t \\ b\sin t \end{pmatrix} \qquad \dot\gamma = \begin{pmatrix} -a\sin t \\ b\cos t \end{pmatrix} \qquad \|\dot\gamma\| = \sqrt{a^2\sin^2 t + b^2\cos^2 t}$$

$$\int_\Gamma F_1(x,y)\,ds = \int_0^{\frac{\pi}{2}} x^2(t)\,\|\dot\gamma\|\,dt = \int a^2\cos^2 t\sqrt{a^2\sin^2 t+b^2\cos^2 t}\,dt$$

$$= \cancel{\int \frac{a^2}{2}(1+\cos 2t)\sqrt{\frac{a^2}{2}}} =$$

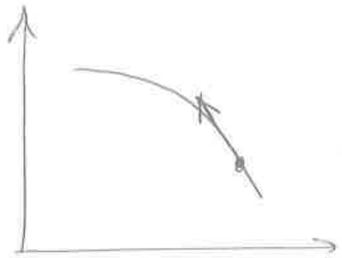
$$= \int a^2\cos^2 t\sqrt{a^2+(b^2-a^2)\cos^2 t}\,dt = \int a^2\cos^2 t\sqrt{(a^2-b^2)\sin^2 t+b^2}\,dt$$

$u = \cos^2 t$ $\qquad$ $u = \sin t$ $\qquad$ $\cos t = \sqrt{1-u^2}$

$du = 2\cos t\sin t\,dt$ $\qquad$ $du = \cos t\,dt$

$$\int a^2\sqrt{\cancel{\cos}-u^2}\sqrt{(a^2-b^2)u^2+b^2}\,du$$

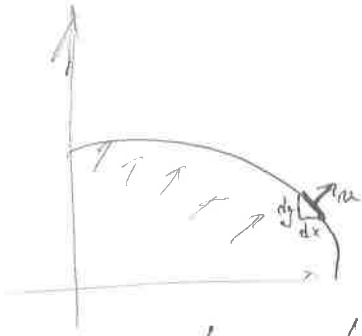
Anal 3
[illegible]



$$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \qquad \gamma(t) = \begin{pmatrix} a\cos t \\ b\sin t \end{pmatrix}$$

$$\dot\gamma(t) = \begin{pmatrix} -a\sin t \\ b\cos t \end{pmatrix}$$

$$\gamma^{\perp}(t) = \begin{pmatrix} y(t) \\ -x(t) \end{pmatrix} \qquad \dot\gamma^{\perp}(t) = \begin{pmatrix} b\cos t \\ a\sin t \end{pmatrix}$$



$$\vec{t} = \begin{pmatrix} dx \\ dy \end{pmatrix}$$

$$F(x,y) = \begin{pmatrix} x^2 \\ y^2 \end{pmatrix} \implies \int_{\Gamma} \langle F(x,y), d\vec{n} \rangle =$$

$$= \int_0^{\frac{\pi}{2}} \langle F(x(t), y(t)), \dot\gamma^{\perp}(t) \rangle \, dt =$$

$$= \int_0^{\frac{\pi}{2}} \left(a^2\cos^2(t)\, b\cos(t) + b^2\sin^2(t)\, a\sin(t) \right) dt =$$

$$= a^2 b \int_0^{\frac{\pi}{2}} \cos^3 t \, dt + b^2 a \int_0^{\frac{\pi}{2}} \sin^3 t \, dt =$$

$$= a^2 b \int \cos t \left(1 - \sin^2 t\right) dt$$

$$\Delta\gamma(t) = \begin{pmatrix} x(t_k) - x(t_{k-1}) \\ y(t_k) - y(t_{k-1}) \end{pmatrix}$$

$$d\vec{n} = \begin{pmatrix} dx \\ -dy \end{pmatrix}$$

$$d\vec{n} = \begin{pmatrix} \dot x(t) \\ \dot y(t) \end{pmatrix} dt$$

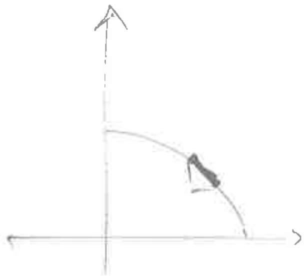
Szenario: $f(x,y) = x^2+y^2$

$\Gamma = \{ (x,y) \in \mathbb{R}^2 \mid (x,y) = \gamma(t), \ldots$

$\Gamma = \{ \gamma(t) \in \mathbb{R}^2 \mid t \in [0, \frac{\pi}{2}] \}$

$$\int_\Gamma f(x,y)\, ds = \int_0^{\frac{\pi}{2}} f(\gamma(t)) \sqrt{\frac{dx^2}{dt^2}+\frac{dy^2}{dt^2}}\, dt$$

$$= \int_0^{\frac{\pi}{2}} f(\gamma(t)) \, \| \dot{\gamma}(t) \| \, dt$$



$$\int f(x)\, dx$$

legen $\cancel{x} = u(t)$

$$dx = \dot{u}(t)\, dt$$

$$\int f(u(t))\, \dot{u}(t)\, dt$$

$$r = \begin{pmatrix} x \\ y \end{pmatrix} = \gamma(t)$$

$$\left.\begin{aligned} ds &= \sqrt{dx^2+dy^2} \\ dx &= \dot{\gamma}_1(t)\, dt \\ dy &= \dot{\gamma}_2(t)\, dt \end{aligned}\right\} \quad ds = \underbrace{\sqrt{\dot{\gamma}_1(t)^2+\dot{\gamma}_2(t)^2}}_{\| \dot{\gamma}(t) \|}\, dt$$

$$\int_\Gamma f(x,y)\, ds = \cancel{\int_\Gamma f(x,y) \sqrt{dx^2+dy^2}}$$

$$= \int f(x(t), y(t)) \sqrt{\cancel{d}x\cancel{(t)}^2+dy^2}$$

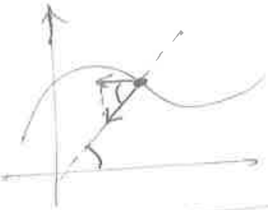
$$x \longmapsto \gamma_1(t)$$

Anal III motivációk:

↳ vektormezők 2D, 3D

↳ gradvektortér

adott $f(x,y) \Rightarrow \nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$



TODO

$\boxed{\text{Vektormezőkre fizikai példák}}$

$-q_1 \quad e \quad +q_2$

$$\vec F = k_e \frac{-q_1 e}{\|\vec r_e - \vec r_{q_1}\|^3}(\vec r_{q_1} - \vec r_e) + k_e \frac{q_2 e}{\|r_{q_2} - r_e\|^3}(\vec r_{q_2} - \vec r_e)$$

↳ lehet több pont is

$r_e = \begin{pmatrix} x \\ y \end{pmatrix} \quad r_{q_1} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$$\vec F_1(x,y) = -k_e \frac{q_1 e \begin{pmatrix} -x + x_1 \\ -y + y_1 \end{pmatrix}}{\left((x_1-x)^2+(y_1-y)^2\right)^{\frac32}}$$

$$= \frac{const}{\left((x-x_1)^2+(y-y_1)^2\right)^{\frac32}}\begin{pmatrix} x - x_1 \\ y - y_1 \end{pmatrix}$$

$$\operatorname{div} \vec F_1(x,y) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} =$$

$$= const\left(-\frac32\right)\left((x-x_1)^2+(y-y_1)^2\right)^{-\frac52}\cdot 2x\cdot(x$$

$+$

$$\sum_{k=0} k_e \frac{\frac{Q}{n} q}{\|r_q - r_k\|^3}(\vec r_k - \vec r_q) =$$

$$r_k = \begin{pmatrix} a + \frac{k}{n}(b-a) \\ 0 \end{pmatrix}, \quad r = \begin{pmatrix} x \\ 0 \end{pmatrix}$$

$$= \sum_{k=0}^{n} k_e \frac{\frac{Q}{b-a} q}{\|r_q - r_k\|^3}(\vec r_k - \vec r_q)\underbrace{\frac{b-a}{n}}_{\Delta x} \to$$

$$\xrightarrow{n\to\infty} k_e \int_a^b \frac{\varrho q}{\left\|r_q - \begin{pmatrix} x \\ 0 \end{pmatrix}\right\|^3}\left(\begin{pmatrix} x \\ 0 \end{pmatrix} - \vec r_q\right) dx$$

adott $f(x,y) = x^2 + 3y \Rightarrow \int_\Gamma f(x,y)\,ds = \int_\Gamma f(x,y)\sqrt{dx^2+dy^2}\cdot\frac{dt}{dt}$

$\gamma(t) = \begin{pmatrix} t \\ 2t \end{pmatrix} \quad t\in[0,1]$

$$= \int_0^1 f(x(t), y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \int_0^1 (t^2 + 6t)\sqrt{1+5}\,dt$$

1/7 a)

$$\mathbb{I} = \int_\Gamma \begin{pmatrix} yz \\ xz \\ xy \end{pmatrix} dr = \int_0^3 \text{polynom-then } dt$$

$$\gamma(t) = \begin{pmatrix} 2t^2 \\ 3t-5 \\ t \end{pmatrix}, t \in [0,3]$$

~~1/8~~

b)

potenciálos-e?

$$\nabla F = \frac{\partial F}{\partial x} + \dots + \frac{\partial F}{\partial z} = 0 \quad \text{potenciálos!}$$

legyen $V : \mathbb{R}^3 \to \mathbb{R}$ s.t. $\text{grad } V = F \Rightarrow$

$$\Rightarrow \frac{\partial V}{\partial x} = F_x \quad \dots \quad \Rightarrow V = \int_{x_0}^{x} F_1(\quad) \dots$$

D_1^* vektorpotenciálos, ha $\exists G : \mathbb{R}^3 \to \mathbb{R}^3$ s.t. $\nabla \times G = F$

$$G(x,y,z) = \int_0^1 t F(tx, ty, tz) \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} dt \quad ; \quad F(x) := \begin{pmatrix} y \\ z \\ x \end{pmatrix}$$

$$= \int_0^1 t \begin{pmatrix} ty \\ tz \\ tx \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} dt = \int_0^1 t \begin{pmatrix} tz^2 - txy \\ tx^2 - tyz \\ ty^2 - txz \end{pmatrix} dt =$$

$$= \frac{1}{3} \begin{pmatrix} z^2 - xy \\ x^2 - yz \\ y^2 - xz \end{pmatrix}$$

$$\begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \frac{1}{3} \begin{pmatrix} z^2 - xy \\ x^2 - yz \\ y^2 - xz \end{pmatrix} = \begin{pmatrix} 2y - y \\ 2z - z \\ 2x - x \end{pmatrix} = \begin{pmatrix} y \\ z \\ x \end{pmatrix}$$

$$\nabla \times (\alpha F + G) = \begin{pmatrix} \frac{\partial}{\partial y}(\alpha F_3 + G_3) - \frac{\partial}{\partial z}(\alpha F_2 - G_2) \\ \frac{\partial}{\partial z}(\alpha F_1 - G_1) - \frac{\partial}{\partial x}(\alpha F_3 - G_3) \\ \end{pmatrix} = \alpha(\nabla \times F) + \nabla \times G$$

ha $\nabla \times F = \nabla \times G$ akkor $F + F_0 = G$

F_0 ~~vektorp~~ skalárpotenciálos!

TODO: Átírni

anal 3
1/7
1/D_1^*

$$\int_\Gamma F(x,y,z)\, d\vec{\ell} = \int_\Gamma \underbrace{f_1(r)\,dx + f_2(r)\,dy + f_3(r)\,dz}_{\omega}$$

$$\cancel{= \int_a^b}$$ leggiu $r = \gamma(t)$ $\qquad \gamma[a,b] = \Gamma$

$$d\vec{\ell} = \dot{\gamma}(t)\,dt$$

$$\int_a^b F(\gamma(t))\,\dot{\gamma}(t)\,dt$$

$$d\ell = \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \dot{\gamma}_1(t)\,dt \\ \dot{\gamma}_2(t)\,dt \\ \dot{\gamma}_3(t)\,dt \end{pmatrix}$$

$$J = \int_C f(x,y)\,d(x,y) = \int_C f(x,y)\,dx \wedge dy$$

leggiu $x = r\cos\theta$, $y = r\sin\theta$ $\qquad \cancel{\Phi(x,y) = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}}$

$$\cancel{D\Phi(x,y)} =$$

$$x = \Phi(r,\theta) = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix} \qquad D\Phi(r,\theta) = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$(r,\theta) \in [0,1]\times[0,2\pi] = I$$

$$\Phi(I) = C$$

$$J = \int_I f(\Phi(r,\theta))\,(D\Phi)\,dr \wedge d\theta$$

$$dx = d\Phi(r,\theta) = \frac{\partial\Phi_1}{\partial r}dr + \frac{\partial\Phi_1}{\partial\theta}d\theta$$

$$dy = d\Phi(r,\theta) = \frac{\partial\Phi_2}{\partial r}dr + \frac{\partial\Phi_2}{\partial\theta}d\theta$$

$$dx \wedge dy = \left(\frac{\partial\Phi_1}{\partial r}dr + \frac{\partial\Phi_1}{\partial\theta}d\theta\right) \wedge \left(\ldots\right) =$$

$$= \frac{\partial\Phi_1}{\partial r}\frac{\partial\Phi_2}{\partial\theta}\,dr\wedge d\theta +$$

$$\frac{\partial\Phi_1}{\partial\theta}\frac{\partial\Phi_2}{\partial r}\,d\theta\wedge dr$$

$$= \left(\frac{\partial\Phi_1}{\partial r}\frac{\partial\Phi_2}{\partial\theta} - \frac{\partial\Phi_1}{\partial\theta}\frac{\partial\Phi_2}{\partial r}\right) dr\wedge d\theta$$

$$D\Phi(r,\theta) = \begin{bmatrix} \frac{\partial\Phi_1}{\partial r} & \frac{\partial\Phi_1}{\partial\theta} \\ \frac{\partial\Phi_2}{\partial r} & \frac{\partial\Phi_2}{\partial\theta} \end{bmatrix} \xrightarrow{\det} |\text{Area} \ldots|$$