

Electromögnesses hullam egyenletek levezetése a Maxwell egyenletekből (váltakban)

A Maxwell egyenletek differenciális alakja:

$$\nabla \cdot \underline{E} = \frac{\rho}{\epsilon_0}$$

váltakban - indukcs áram
(nem folyócs áram)
- indukcs töltség

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

vagyis $\underline{J} = \underline{0}$

$$\nabla \times \underline{B} = \mu_0 \left(\underline{J} + \epsilon_0 \frac{\partial \underline{E}}{\partial t} \right)$$

$$\underline{J} = \underline{0}$$

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$$\nabla \cdot \underline{E} = 0$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \underline{B} = \mu_0 \epsilon_0 \frac{\partial \underline{E}}{\partial t}$$

Ismeret összefüggés:

$$\underline{u} \times (\underline{v} \times \underline{w}) = \underline{u} \langle \underline{v}, \underline{w} \rangle - \langle \underline{u}, \underline{v} \rangle \underline{w}$$

$$\text{Így } \nabla \times (\nabla \times \underline{E}) = \nabla (\nabla \cdot \underline{E}) - (\nabla \cdot \nabla) \underline{E} = -\Delta \underline{E}$$

Lehet $\Delta \underline{E} = -\nabla \times (\nabla \times \underline{E}) = +\nabla \times \frac{\partial \underline{B}}{\partial t} = \frac{\partial}{\partial t} (\nabla \times \underline{B}) \Rightarrow$

$$\Rightarrow \Delta \underline{E} = \frac{\partial}{\partial t} \mu_0 \epsilon_0 \frac{\partial \underline{E}}{\partial t}$$

~~$\frac{1}{c^2}$~~ $\Delta \underline{E} = \frac{\partial^2 \underline{E}}{\partial t^2} \frac{1}{c^2}$ ahol ~~$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$~~ $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$\epsilon_0 \approx 8.85 \cdot 10^{-12} \frac{\text{F}}{\text{m}}$$

$$\mu_0 \approx 4\pi \cdot 10^{-7} \approx 1.25 \cdot 10^{-6} \frac{\text{H}}{\text{m}}$$

$$c = \sqrt{\mu_0 \epsilon_0}^{-1} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 300.000 \frac{\text{km}}{\text{s}} = 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

13. gyök.

PDE hullameg
Levezetés Maxwell